Lagrangian Coherent Structures and their Uncertainties in the Monterey Bay and California Current System region

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Main collaborators on this topic

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AOSN2-MURI: **N. Leonard, J. Marsden, S. Ramp, R. Davis, D. Fratantoni**

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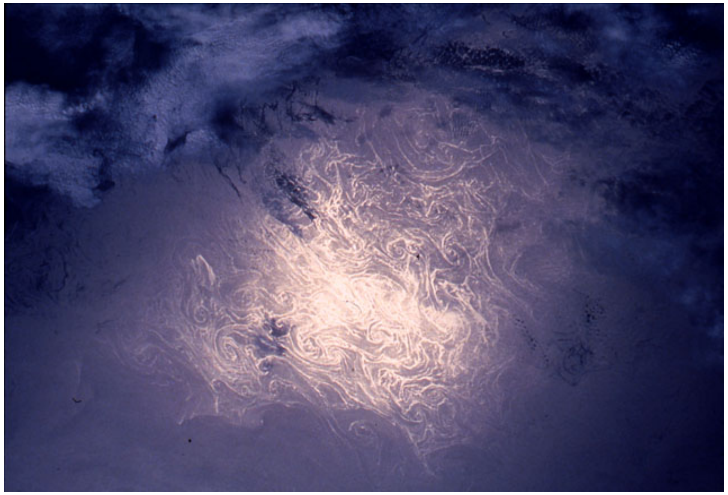
1. **Introduction and Concepts: Coherent Structures in the Ocean and Uncertainties**
2. **Dynamical and Uncertainty equations**
3. **Uncertainty Algorithms and Computational Systems: HOPS, ESSE, MANGEN**
4. **Results: Uncertainty predictions for 3 dynamical events in the Monterey Bay region**
5. **Conclusions**

MIT-ME Mechanics Seminar Series December 5, 2005

Evidence of ‘Coherent’ Structures in the Ocean: Shuttle Photographs



Gulf Stream, Spiral Eddies

Spiral eddies (12 to 18 kms in diameter) in the Gulf Stream (1984) highlighted in Sun glitter. When spiral eddies were first observed in the Gulf of Oman (first shuttle mission, 1981), some thought that sub-mesoscale eddies were perhaps unique to that region.

Greek Island, Spiral Eddies and Wakes

Eastern end of Crete and some of the smaller Greek islands. Extensive spiral eddy field in the Sun glitter in the center of the frame, as well as island wakes — a phenomenon created where islands interrupt the flow pattern or ocean water.

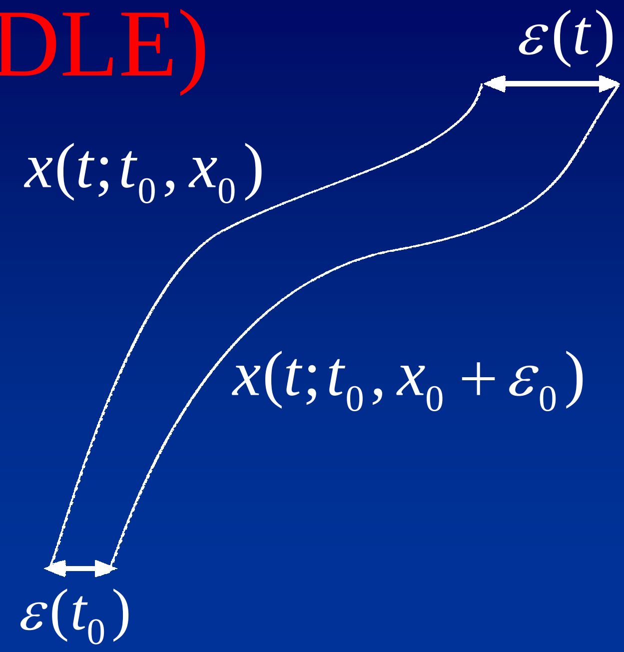
Gulf Stream, Seasonal Plankton Bloom

Gulf Stream shear zone and associated cold core eddies (1985).

Discovery of "coherent structures" in turbulent fluid flow has been an important advance in fluid mechanics.

A "coherent structure" may be thought of as a shape or form in a turbulent fluid flow that persists a long time relative to it's own period of internal circulation.

Direct Lyapunov Exponent (DLE)



The finite-time Lyapunov exponent is the maximum exponential growth about a trajectory:

$$\sigma_T(t_0,\mathbf{x}_0) = \frac{1}{T}\ln \max_{\epsilon(t_0)} \left\{ \frac{\epsilon(t)}{\epsilon(t_0)} \right\}$$

More precisely:

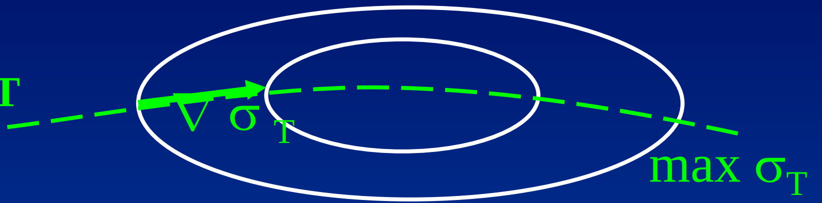
$$\sigma_T = \frac{1}{T}\ln \lambda_{\max} \left\{ \frac{\partial \mathbf{x}(\mathbf{t_0{+}T},\mathbf{t_0},\mathbf{x_0})}{\partial \mathbf{x}_0}^{\top} \frac{\partial \mathbf{x}(\mathbf{t_0{+}T};\mathbf{t_0},\mathbf{x_0})}{\partial \mathbf{x}_0} \right\}$$

The DLE, **Haller (2001)**, is the finite-time LE computed *directly*, using a set of particles released for duration T in a numerical simulation, fluid flow, etc.

The (D)-LE is also the normalized log of the maximum singular value of the finite time strain tensor dx/dx_0. Of course, $x \in R^2$ here!

Lagrangian Coherent Structures

LCS is approximated as a ridge of σ_T



1. LCS is parallel to $\nabla \sigma_T$
2. On the LCS, Hessian $\Sigma_T = \nabla^2 \sigma_T$ is negative and minimum in the direction normal to the LCS (i.e., maximum curvature).

Lagrangian coherent structures indicate high stretching and the presence of hyperbolic trajectories (i.e., trajectories with stable and unstable manifolds),

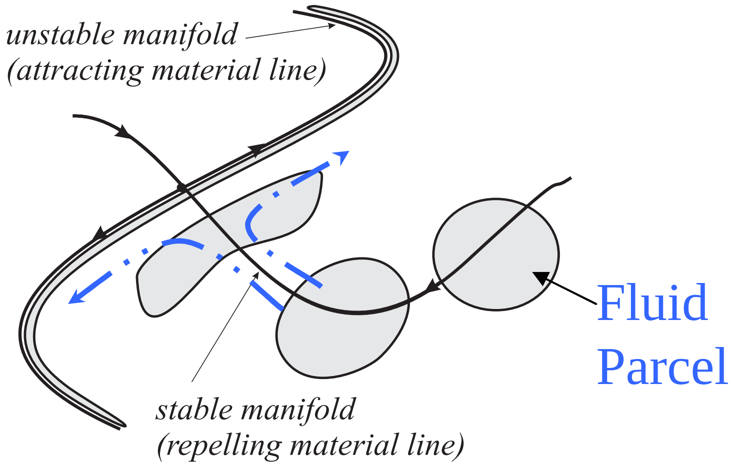
[Haller 2001]

Lagrangian coherent structures are almost invariant manifolds, hence they are a "good approximation" of stable and unstable manifolds.

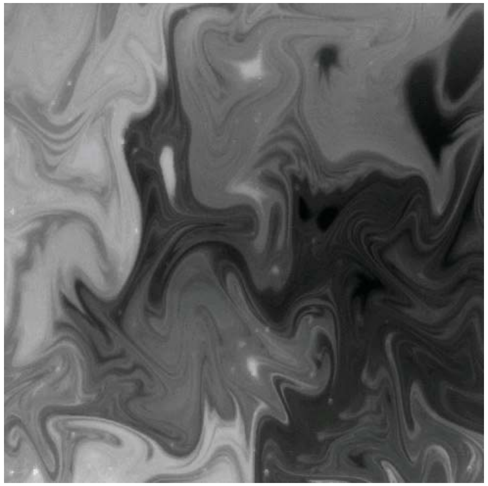
[Shadden, Lekien and Marsden 2001]

Evidence of LCS transport

We refer to a stable manifold as a Repelling Material Line since it would tend to stretch a parcel of Lagrangian particles placed about it, whereas the unstable manifold is deemed an Attracting Material Line since the parcel would get attracted to it as shown to the right.



Mixing of colored dyes.



Left: Picture of fluid with dye (lab experiment)

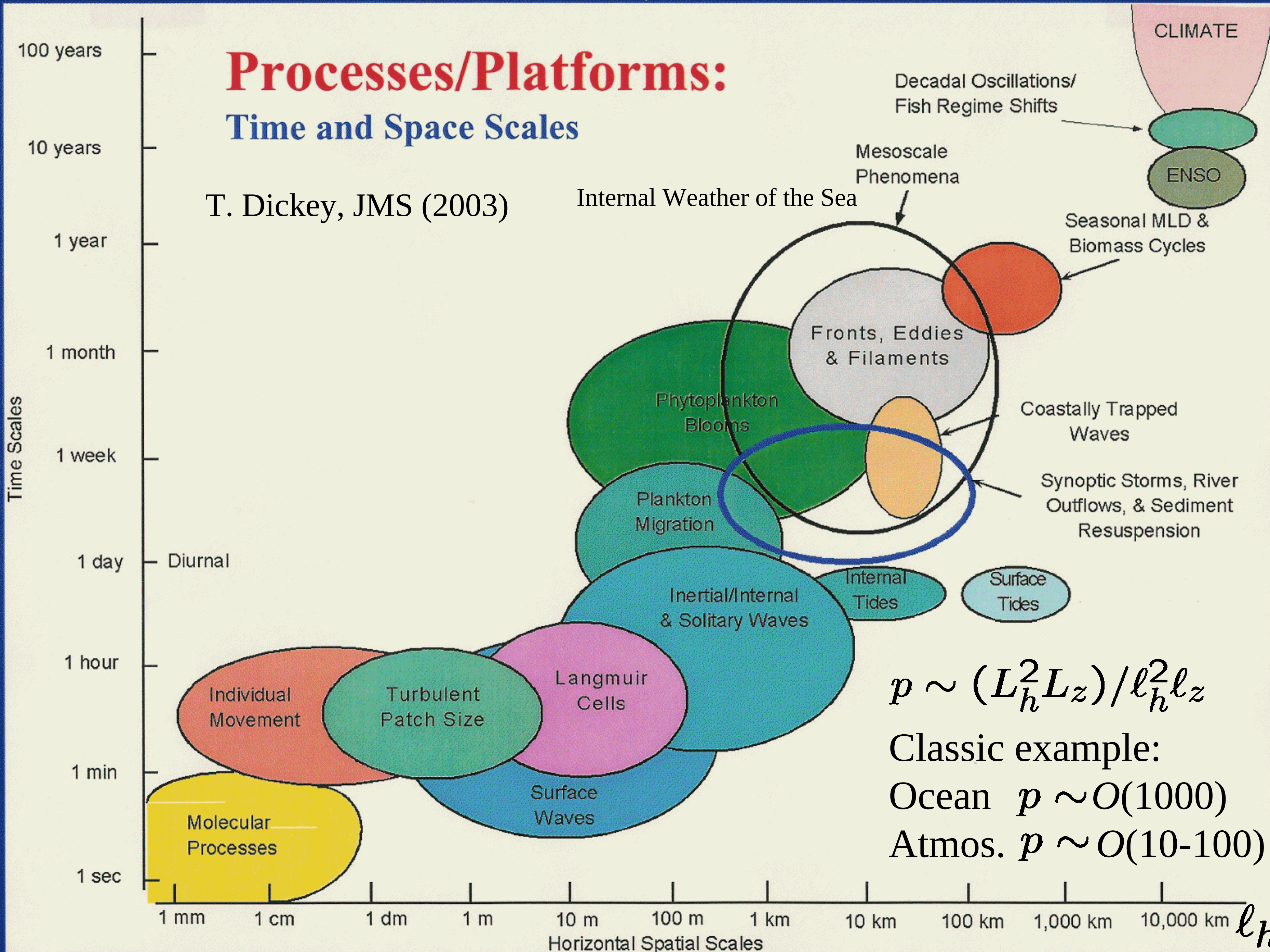
Right: DLE contour (computed from u,v data) superimposed. Ridges of high DLE are shown in red.

Voth, Haller, & Gollub [PRL 18 (2002)

Processes/Platforms:

Time and Space Scales

T. Dickey, JMS (2003)



$$p \sim (L_h^2 L_z)/\ell_h^2 \ell_z$$

Classic example:

Ocean $p \sim O(1000)$

Atmos. $p \sim O(10\text{-}100)$

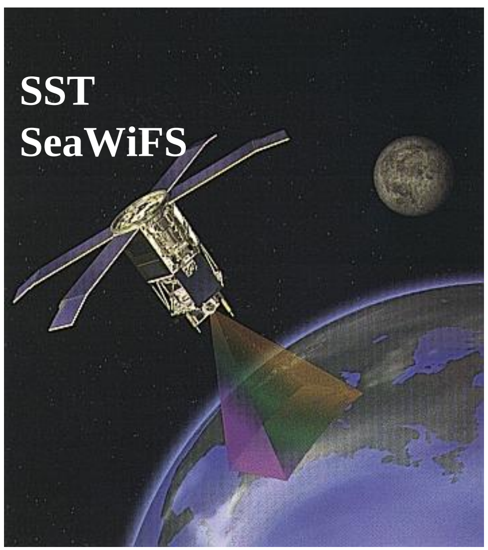
Physical and Multidisciplinary Observations

AUV

Aircraft

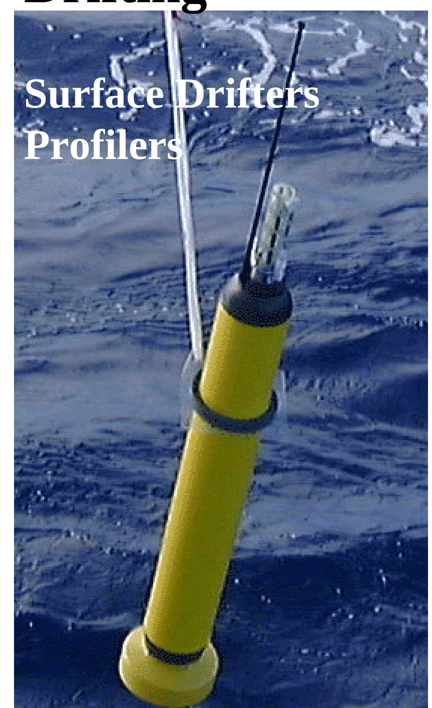
Ships

Satellite



Moored/Fixed

Drifting



Geophysical Fluid Dynamics (GFD)

Study of geophysical flows and dynamics (Earth atmosphere, ocean, etc)

Fundamental equations are Navier-Stokes in rotating frame of reference

Additional practical assumptions limit the range of modeled scales in time and space. The assumptions used here are:

1. Boussinesq fluid (small variations of density about a state of reference)
2. Turbulent flow reduced to scale window of interest, here:
   - (Sub)-mesoscale to large-scale ocean processes
   - Processes outside this window are averaged and their effects parameterized (turbulent closures)
3. Thinness approximation ($H/L \ll 1$)

Result: the so-called **Primitive-Equations of Ocean Dynamics**

Models of (Interdisciplinary) Ocean Dynamics Utilized

- **Physical model:** **Primitive-Equation (PDE, x, y, z, t: HOPS)**

Horiz. Mom.	$\frac{D\mathbf{u_h}}{Dt} + f\,\mathbf{e}_3 \wedge \mathbf{u}_h = -\frac{1}{\rho_0}\nabla_h\, p_w + \nabla_h \cdot (A_h \nabla_h \mathbf{u}_h) + \frac{\partial\; A_v\; \partial \mathbf{u}_h/\partial z}{\partial z}$	**(1-2)**
Vert. Mom.	$\rho g + \frac{\partial p_w}{\partial z} = 0$	**(3)**
Thermal en.	$\frac{DT}{Dt} = \nabla_h \cdot (K_h \nabla_h T) + \frac{\partial\; K_v\; \partial T/\partial z}{\partial z}$	**(4)**
Cons. of salt	$\frac{DS}{Dt} = \nabla_h \cdot (K_h \nabla_h S) + \frac{\partial\; K_v\; \partial S/\partial z}{\partial z}$	**(5)**
Cons. of mass	$\nabla \cdot \mathbf{u} = 0$	**(6)**
Eqn. of state	$\rho(\mathbf{r}, z, t) = \rho(T, S, p_w)$	**(7)**

- **Biogeochemical model:** **Generic ADR equation (PDE, x, y, z, t)**

$$\frac{\partial \phi_i}{\partial t} + \mathbf{u} \cdot \nabla \phi_i - \nabla_h (A_i \nabla_h \phi_i) - \frac{\partial\; K_i\, \partial \phi_i/\partial z}{\partial z} = \mathcal{B}_i(\phi_1, \ldots, \phi_i, \ldots, \phi_7) \qquad (\mathbf{8-14})$$

$i = \mathrm{NO3},\ \mathrm{P}_{NO3},\ \mathrm{ZOO},\ \mathrm{NH4},\ \mathrm{DET},\ \mathrm{CHL},\ \mathrm{P}_{NH4}$

DEFINITION AND REPRESENTATION OF UNCERTAINTY

- $\boldsymbol{x}$ = estimate of some quantity (measured, predicted, calculated)

- $\boldsymbol{x}^t$ = actual value (unknown true nature)

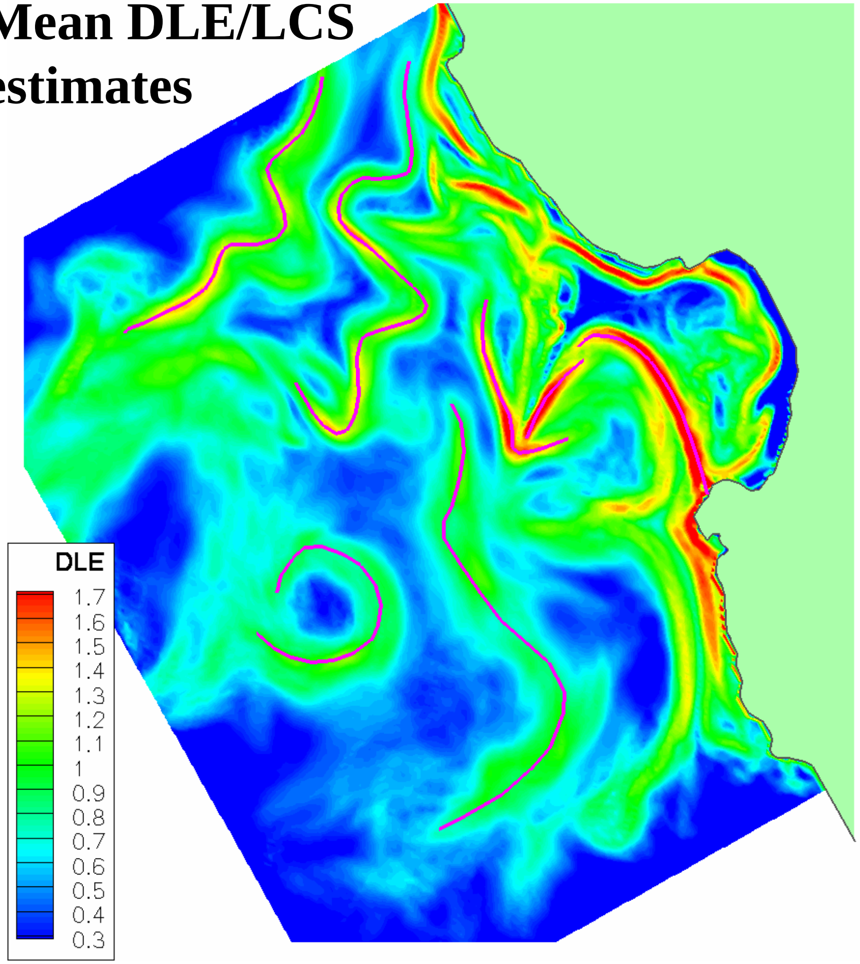
- $\boldsymbol{e} = \boldsymbol{x} - \boldsymbol{x}^t$ (unknown error)

Uncertainty in $\boldsymbol{x}$ is a representation of the error estimate $\boldsymbol{e}$
e.g. probability distribution function of $\boldsymbol{e}$

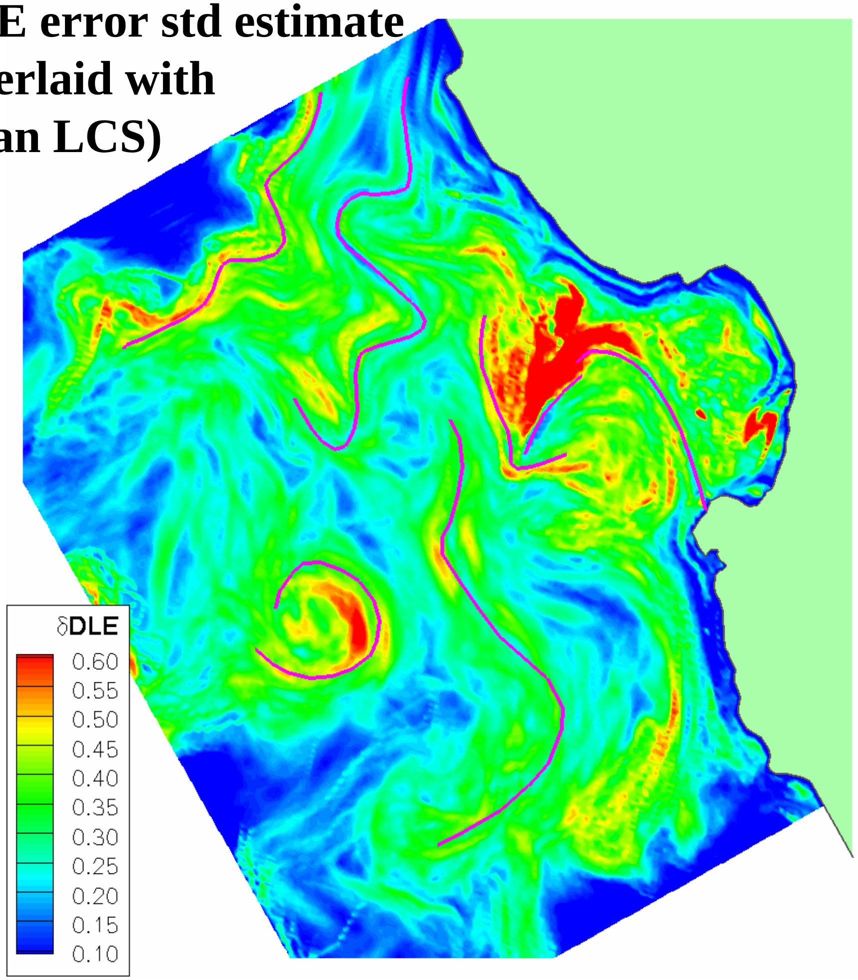
- Variability in $\boldsymbol{x}$ vs. Uncertainty in $\boldsymbol{x}$
- Uncertainties in general have structures, in time and in space: They can be represented as fields

One of Main Goals here:
Estimate and Study Uncertainties of LCSs

Mean DLE/LCS estimates



DLE error std estimate (overlaid with mean LCS)



How to compute such uncertainty fields?

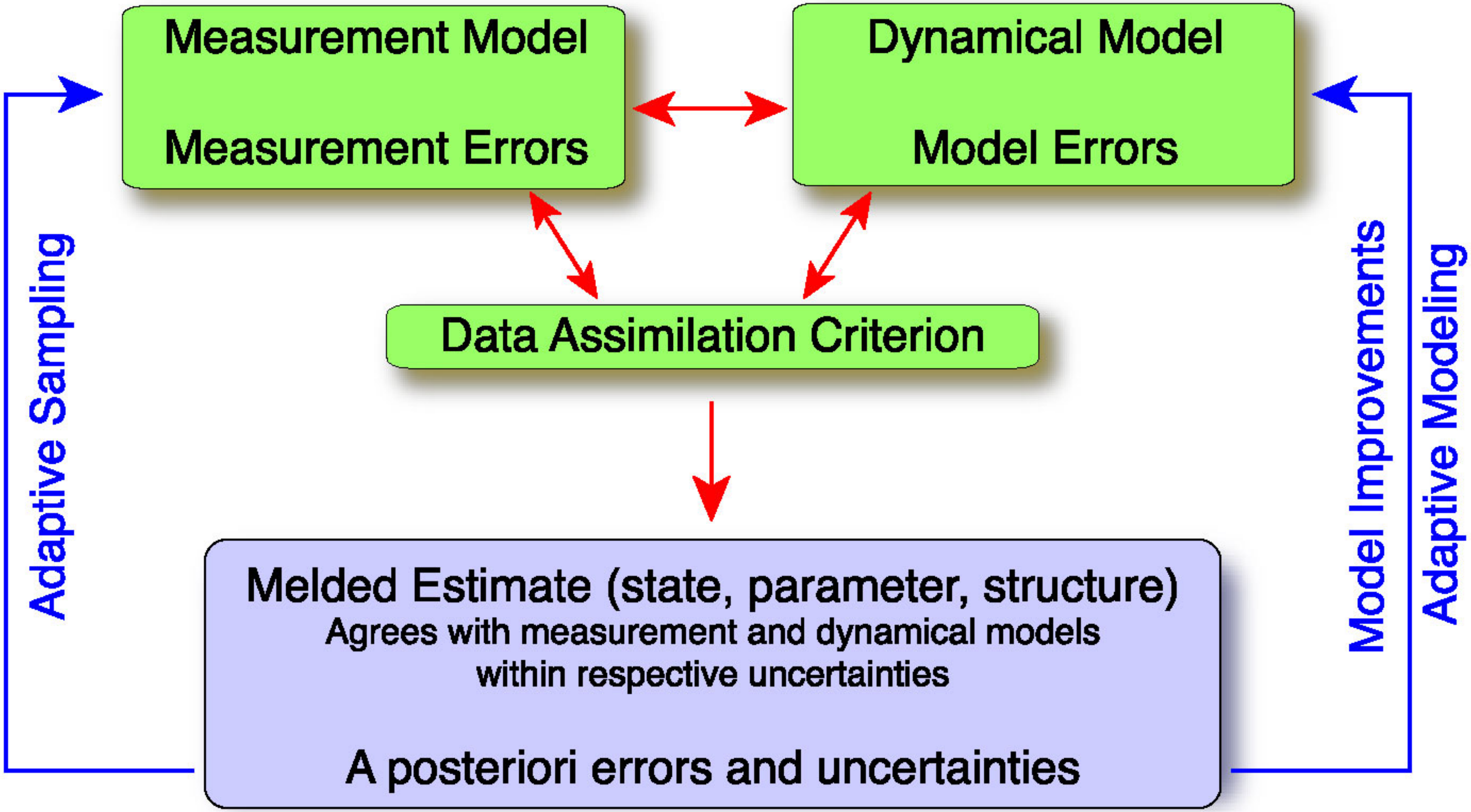
MAIN SOURCES OF UNCERTAINTIES IN MODERN OCEAN SCIENCE

- **Ocean Physics model uncertainties**
  - Bathymetry
  - Initial conditions
  - BCs: surface atmospheric, coastal-estuary and open-boundary fluxes
  - Parameterized processes: sub-grid-scales, turbulence closures, un-resolved processes
    - e.g. tides and internal tides, internal waves and solitons, microstructure and turbulence
  - Numerical errors: steep topographies/pressure gradient, non-convergence

- **Measurement uncertainties**
  - Sensor errors (random and bias/drift)
  - Environmental noise (processes measured but not of interest, e.g. of scales outside of studied scale window)
  - Equation(s) linking model variables to measured variables

WHAT IS DATA ASSIMILATION?

A Melded Estimate of Data and Dynamics



e.g. Robinson A.R., P.F.J. Lermusiaux and N.Q. Sloan, III (1998). *Data Assimilation*. The Sea, Vol. 10.

Robinson A.R. and P.F.J. Lermusiaux (2002). *DA for physical-biological interactions*. The Sea, Vol.12.

Generic Data Assimilation (DA) Problem

Dynamical models:

$$d\phi_i + \mathbf{u} \cdot \nabla\phi_i dt - \nabla(K_i \nabla\phi_i)dt = B_i(\phi_1, \ldots, \phi_i, \ldots, \phi_n)dt + d\eta_i \qquad (i = 1, \cdots, n)$$

e.g. i= u, v, T, $\cdots$, ZOO, $\cdots$, p

Parameter equations:

$$dP_\ell = C_\ell(\phi_1, \ldots, \phi_i, \ldots, \phi_n)dt + d\zeta_\ell \qquad (\ell = 1, \cdots, p)$$

e.g. $P_\ell = \{\ K_i,\ R_i, \cdots\}$

Measurement models:

$$y_j = \mathcal{H}_j(\phi_1, \cdots, \phi_i, \cdots, \phi_n) + \epsilon_j \qquad (j = 1, \cdots, m)$$

e.g. $y_j = \{\ XBT_j,\ Fluo_j,\ SSH_j,\ CODAR_j\}$

Assimilation criterion:

$$\min_{\phi_i, P_\ell} \quad J(d\eta_i\ ,\ d\zeta_\ell\ ,\ \epsilon_j\ ,\ q_\eta\ ,\ q_\zeta\ ,\ q_\epsilon)$$

CLASSES OF DATA ASSIMILATION SCHEMES

	Error Evol.	Criterion
• **Estimation Theory (Filtering and Smoothing)**		
1. Direct Insertion, Blending, Nudging	- Linear	
2. Optimal interpolation	- Linear	LS
3. Kalman filter/smoother	- Linear	LS
4. Bayesian estimation (Fokker-Plank equations)	- Non-lin.	Non-LS
5. Ensemble/Monte-Carlo methods	- Non-lin.	LS/Non-LS
6. Error-subspace/Reduced-order methods: Square-root filters, e.g. SEEK	- (Non)-Lin.	LS
7. Error Subspace Statistical Estimation (ESSE): 5 and 6	-Non-lin.	LS/Non-LS
• **Control Theory/Calculus of Variations (Smoothing)**		
1. “Adjoint methods” (+ descent)	- Lin. adj.	LS
2. Generalized inverse (e.g. Representer method + descent)	- Lin. adj.	LS
• **Optimization Theory (Direct local/global smoothing)**		
1. Descent methods (Conjugate gradient, Quasi-Newton, etc)	- Lin	LS/Non-LS
2. Simulated annealing, Genetic algorithms	- Non-lin.	LS/Non-LS

- **Hybrid Schemes**
  - Combinations of the above

Ocean and LCS Estimation Problem

Continuous Problem

Ocean Dynamical Model

$$d\phi_i + \mathbf{u}.\nabla\phi_i dt = \nabla(K_i\nabla\phi_i)dt + d\eta_i \quad (\phi_i = u, v, \psi, T, S)$$

Measurement Model

$$y_j = \mathcal{H}_j(\phi_1, \cdots, \phi_i, \cdots, \phi_n) + \epsilon_j \quad (y_j = T_j, S_j)$$

Assimilation Criterion

$$\min_{\phi_i} \quad J(\phi_i, y_j \,;\, d\eta_i\,,\, d\zeta_\ell\,,\, \epsilon_j\,,\, q_\eta\,,\, q_\zeta\,,\, q_\epsilon)$$

Particles Trajectory

$$d\mathbf{x} = \mathbf{v}(\mathbf{x}, t; \eta_i, \epsilon_j)\ dt$$

DLE/LCS computation LCS is defined as a ridge of $\sigma_T(x,y,z,t)$

Continuous-Discrete Problem

On discrete grid of ocean model **M**, define state vector: $\mathbf{x} = (\mathbf{u}, \mathbf{v}, \psi, \mathbf{T}, \mathbf{S})$

$$d\mathbf{x} = \mathrm{M}(\mathbf{x}, x, y, z, t)dt + d\eta(z) \qquad \eta \sim \mathcal{N}(0, \mathbf{Q})$$

$$\mathbf{y} = \mathbf{H}_j(\mathbf{x}, x, y, z, t)dt + \epsilon(z) \qquad \epsilon \sim \mathcal{N}(0, \mathbf{R})$$

$$\min_{\mathbf{x}}\ J(\mathbf{x}, \mathbf{y}_j\,;\, d\eta\,,\, \epsilon\,,\, \mathbf{Q}\,,\, \mathbf{R})$$

$$d\chi = \mathbf{v}(x, y, z, t)dt$$

LCS defined as above

Evolution equations for PDF (Continuous-Discrete Problem)

- Fokker-Planck equation (prediction) and Bayes' rule at observation times (DA):

$$\frac{\partial p(\mathbf{x},t\,|\,\mathbf{y}_{t-})}{\partial t} = -\sum_{i=1}^{n}\frac{\partial\left(p(\mathbf{x},t\,|\,\mathbf{y}_{t-})\mathcal{M}_i(\mathbf{x},t)\right)}{\partial \mathbf{x}_i} + \frac{1}{2}\sum_{i,j=1}^{n}\frac{\partial^2\left(p(\mathbf{x},t\,|\,\mathbf{y}_{t-})\mathbf{Q}_{ij}\right)}{\partial \mathbf{x}_i\partial \mathbf{x}_j} \tag{3a}$$

$$p(\mathbf{x},t_k\,|\,\mathbf{y}_0^o,\ldots,\mathbf{y}_k^o) = \frac{p(\mathbf{y}_k^o\,|\,\mathbf{x})\;p(\mathbf{x},t_k\,|\,\mathbf{y}_0^o,\ldots,\mathbf{y}_{k-1}^o)}{\int p(\mathbf{y}_k^o\,|\,\chi)\;p(\chi,t_k\,|\,\mathbf{y}_0^o,\ldots,\mathbf{y}_{k-1}^o)\;d\chi} \tag{3b}$$

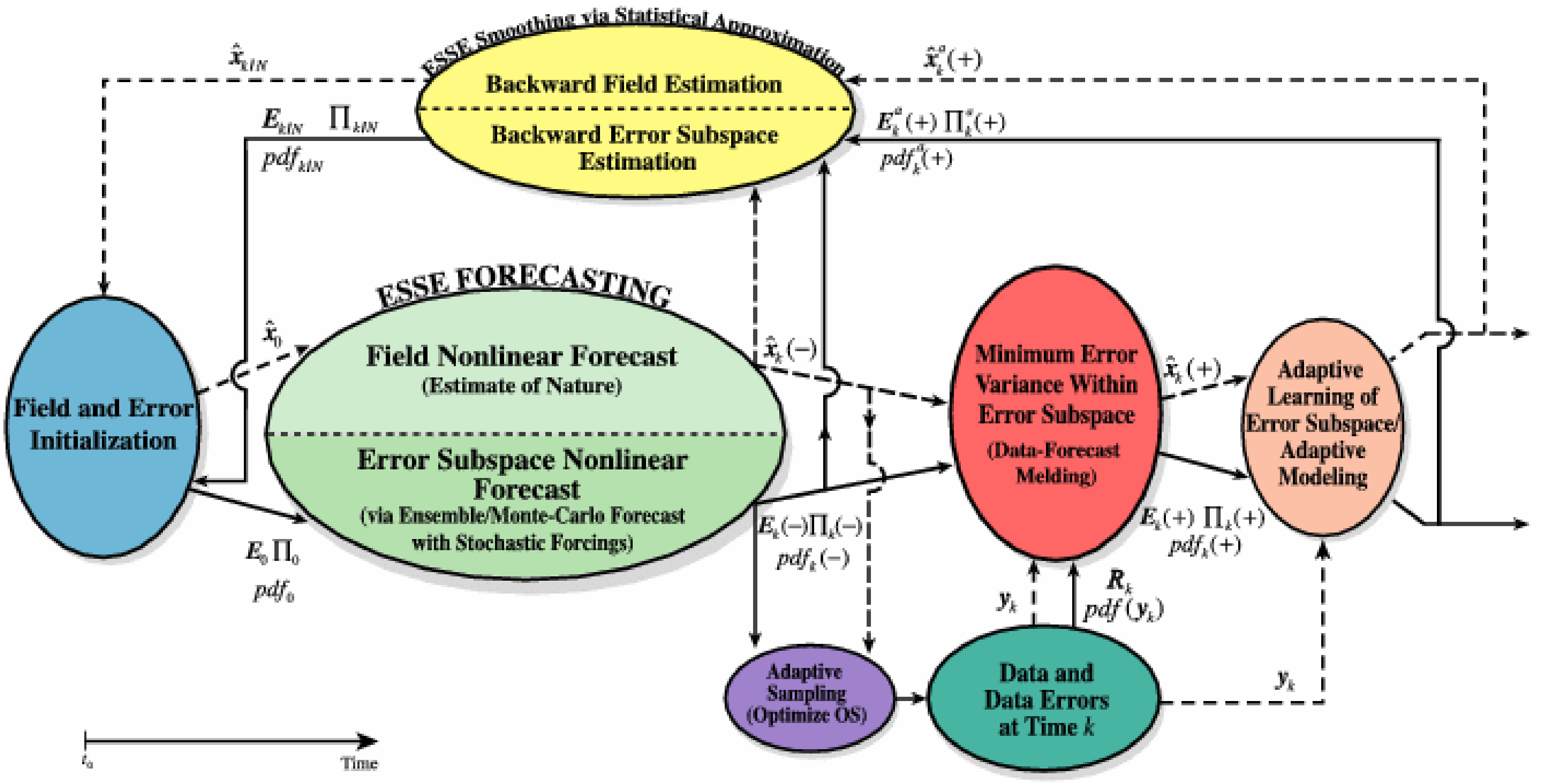
- Realistic ocean applications focus on conditional mean and error covariance matrix:

$$\frac{d\mathrm{P}}{dt} = <(\mathbf{x}-\widehat{\mathbf{x}})(\mathcal{M}(\mathbf{x})-\mathcal{M}(\widehat{\mathbf{x}}))^T> + <(\mathcal{M}(\mathbf{x})-\mathcal{M}(\widehat{\mathbf{x}}))(\mathbf{x}-\widehat{\mathbf{x}})^T> + \mathbf{Q} \tag{4a}$$

$$\mathrm{P}_k(+) = \frac{<\mathbf{x}_k\,\mathbf{x}_k^T\,p(\mathbf{y}_k^o\,|\,\mathbf{x}_k)>_-}{<p(\mathbf{y}_k^o\,|\,\mathbf{x}_k)>_-} - \widehat{\mathbf{x}}_k(+)\,\widehat{\mathbf{x}}_k(+)^T \tag{4b}$$

- Four Sources/Sinks of uncertainties:
  1. Initial Conditions: **P**(0)
  2. Deterministic dynamics (including nonlinear terms in PE equations)
  3. Stochastic forcings (model uncertainties which increase variance)
  4. DA which reduces variance: data type, locations and uncertainties (Zakai eqn)

Error Subspace Statistical Estimation (ESSE)



- Ensemble-based (with nonlinear and stochastic PE model of HOPS)
- Uncertainty forecasts (with dynamic ES, convergence criteria, error learning)
- Multivariate, non-homogeneous and non-isotropic DA
- Consistent DA and adaptive sampling schemes
- Software: not tied to any model, but specifics currently tailored to HOPS

Regions and Field Exercises for which ESSE has been utilized

- Strait of Sicily (AIS96-RR96), Summer 1996
- Ionian Sea (RR97), Fall 1997
- Gulf of Cadiz (RR98), Spring 1998
- Massachusetts Bay (LOOPS), Fall 1998
- Georges Bank (AFMIS), Spring 2000
- Massachusetts Bay (ASCOT-01), Spring 2001
- Middle Atlantic Bight shelfbreak front region (hindcast for PRIMER-06)
- Monterey Bay (AOSN-2), Summer 2003
- Lagoon of Venice (with OGS Trieste, hindcast for whole 2001)
- Elba-Pianosa region in Ligurian Sea (FAF05), Summer 2005

For publications, email me or see http://www.deas.harvard.edu/~pierrel

e.g. Recent invited manuscripts in 3 special issues:

J. Comp. Phys. (2006), Oceanography magazine (2006), Physica D (2006)

Stochastic Forcing in Ocean Modeling

There are essentially three approaches

1. **Empirical**: uses the misfits between deterministic model forecasts and observations, and organizes and maps these misfits back to the state space (e.g. Dee, 1995), possibly using a dominant SVD.
2. **Analytical**: derives stochastic equations for the most energetic deficient processes of the dynamical model (PE).
3. **Numerical**‘: utilizes notions related to stochastic optimals, (e.g. Farrell and Ioannou, 1994) to estimate what I call “model error optimals”.

Presently, we utilize: a simple, zeroth order version of the analytical approach, with model coefficients empirically estimated from observations.

Our Stochastic Forcing Model

- Aims to represent statistical effects of sub-grid-scales
  (e.g. sub-mesoscales, internal tides, etc)
- Is correlated in time and space
- Amplitudes set to "ε x ||geostrophy(z)||"

1. Temporal correlations (for a scalar process)

I. 0d Random Noise Exponentially Decorrelated in Time

$$d\tilde{w} + \beta\,\tilde{w}\,dt = dw, \tag{76}$$

$$\dot{p}_{\tilde{w}} = -2\beta\,p_{\tilde{w}} + q\ . \tag{77}$$

Setting $\dot{p}_{\tilde{w}}$ to zero at all times yields $p_{\tilde{w}}(0) = \sigma^2 = \frac{q}{2\beta}$. The process $\tilde{w}$ is assumed to be of fixed fluctuation amplitude σ and autocorrelation time $\frac{1}{\beta}$. The constant variance of the white noise w is thus set to $q = 2\beta\sigma^2$.

2. Spatial correlations

II. 3d Random Noise, Exponentially Decorrelated in Time and 2-Grid Point Decorrelated in Space

$$d\mathbf{x} = \mathcal{M}(\mathbf{x}, t)\; dt + \mathrm{B}^{fc}(t)\; d\tilde{\mathbf{w}}^c$$
$$d\tilde{\mathbf{w}}^c = -\beta^c \tilde{\mathbf{w}}^c\, dt + d\mathbf{w}^c\ ,$$

where symbols denote the:

- **discrete-space PE state vector:** $\mathbf{x} = (\hat{\mathbf{u}}, \hat{\mathbf{v}}, \mathbf{T}, \mathbf{S}, \mathbf{p})^T \in \mathbb{R}^n$
- **coarse 3d white noise:** $\mathbf{w}^c$
- **Coarse 3d Gauss-Markov process:** $\tilde{\mathbf{w}}^c$

  i.e. $\mathbf{dw}^c = (\mathbf{dw}^c_{\hat{u}}, \mathbf{dw}^c_{\hat{v}}, \mathbf{dw}^c_{T}, \mathbf{dw}^c_{S}, \mathbf{dw}^c_{\psi})^T$
- **PE dynamical model operator:** $\mathcal{M}(\mathbf{x}, t)$
- **linear extrapolation matrix, from coarse to fine state:** $\mathbf{B}^{fc}(t)$

Stochastic Primitive Equation Model

The diagonal of time-decorrelations:

$\beta_u, \beta_v, \beta_T, \beta_S, \beta_\psi$ **functions of** (x, y, z)

are here chosen $\beta_X = \beta \mathbf{I}$.

The diagonal of noise variances are chosen function of ***z*** **only, of amplitude set to: "ε * geostrophy"**

$$\boldsymbol{\Sigma}_u = \boldsymbol{\Sigma}_v = \sigma^2(z)\,\mathbf{I}\ , \quad \text{with } \sigma_U(z) = \epsilon_U\, f_c\, U(z)\ ,$$

$$\boldsymbol{\Sigma}_T = \sigma_T^2(z)\,\mathbf{I}\ , \quad \text{with } \sigma_T(z) = \epsilon_T\, U(z)\, \frac{\Delta T(z)}{L(z)}\ ,$$

$$\boldsymbol{\Sigma}_S = \sigma_S^2(z)\,\mathbf{I}\ , \quad \text{with } \sigma_S(z) = \epsilon_S\, U(z)\, \frac{\Delta S(z)}{L(z)}\ ,$$

$$\boldsymbol{\Sigma}_\psi = \sigma_\psi^2(z)\,\mathbf{I}\ , \quad \text{with } \sigma_\psi(z) = \epsilon_\psi\, \frac{\varpi\, L(z)}{U(z)}\ ,$$

Internal Baroclinic Zonal Mode

$$\mathbf{d\hat{u}} = \mathbf{du'} - \mathbf{d\overline{u}'}\ ,$$

$$\mathbf{du'} = \left(-\boldsymbol{\Gamma}(\mathbf{u}) + \mathbf{fv} - \frac{\mathbf{g}}{\boldsymbol{\rho}_0}\int_{\mathbf{z}}^{0} \boldsymbol{\rho}_{\mathbf{x}}\,\mathbf{dz} + \mathbf{F}_u + \mathbf{A}_v\,\mathbf{u_{zz}}\right) dt + \mathbf{B}_u^{fc}\,\mathbf{d\tilde{w}}_u^c\ ,$$

$$\text{with} \quad \mathbf{u} = \hat{\mathbf{u}} - \frac{1}{\mathbf{H}}\boldsymbol{\psi}_{\mathbf{y}}\ .$$

$$\mathbf{d\tilde{w}}_u^c = -\boldsymbol{\beta}_u\,\tilde{\mathbf{w}}_u^c\,dt + \mathbf{dw}_u^c\ ,$$

$$\text{with} \quad \tilde{\mathbf{w}}_u^c(0) \sim (\mathbf{0}, \boldsymbol{\Sigma}_u) \quad \text{and} \quad \mathbf{w}_u^c \sim (\mathbf{0}, 2\boldsymbol{\beta}_u\,\boldsymbol{\Sigma}_u)\ .$$

Internal Baroclinic Meridional Mode

$$\mathbf{d\hat{v}} = \mathbf{dv'} - \mathbf{d\overline{v}'}\ ,$$

$$\mathbf{dv}'_t = \left(-\boldsymbol{\Gamma}(\mathbf{v}) - \mathbf{fu} - \frac{\mathbf{g}}{\boldsymbol{\rho}_0}\int_{\mathbf{z}}^{0} \boldsymbol{\rho}_y\,\mathbf{dz} + \mathbf{F}_v + \mathbf{A}_v\,\mathbf{v_{zz}}\right) dt + \mathbf{B}_v^{fc}\,\mathbf{d\tilde{w}}_v^c\ ,$$

$$\text{with} \quad \mathbf{v} = \hat{\mathbf{v}} + \frac{1}{\mathbf{H}}\boldsymbol{\psi}_{\mathbf{x}}\ .$$

$$\mathbf{d\tilde{w}}_v^c = -\boldsymbol{\beta}_v\,\tilde{\mathbf{w}}_v^c\,dt + \mathbf{dw}_v^c\ ,$$

$$\text{with} \quad \tilde{\mathbf{w}}_v^c(0) \sim (\mathbf{0}, \boldsymbol{\Sigma}_v) \quad \text{and} \quad \mathbf{w}_v^c \sim (\mathbf{0}, 2\boldsymbol{\beta}_v\,\boldsymbol{\Sigma}_v)\ .$$

Thermal energy: Balance

$$\mathbf{dT} = \left(-\boldsymbol{\Gamma}(\mathbf{T}) + \mathbf{F}_T + \mathbf{K}_v\,\mathbf{T_{zz}}\right) dt + \mathbf{B}_T^{fc}\,\mathbf{d\tilde{w}}_T^c\ ,$$

$$\mathbf{d\tilde{w}}_T^c = -\boldsymbol{\beta}_T\,\tilde{\mathbf{w}}_T^c\,dt + \mathbf{dw}_T^c\ ,$$

$$\text{with} \quad \tilde{\mathbf{w}}_T^c(0) \sim (\mathbf{0}, \boldsymbol{\Sigma}_T) \quad \text{and} \quad \mathbf{w}_T^c \sim (\mathbf{0}, 2\boldsymbol{\beta}_T\,\boldsymbol{\Sigma}_T)\ .$$

Conservation : of Salt

$$\mathbf{dS} = \left(-\boldsymbol{\Gamma}(\mathbf{S}) + \mathbf{F}_S + \mathbf{K}_v\,\mathbf{S_{zz}}\right) dt + \mathbf{B}_S^{fc}\,\mathbf{d\tilde{w}}_S^c\ ,$$

$$\mathbf{d\tilde{w}}_S^c = -\boldsymbol{\beta}_S\,\tilde{\mathbf{w}}_S^c\,dt + \mathbf{dw}_S^c\ ,$$

$$\text{with} \quad \tilde{\mathbf{w}}_S^c(0) \sim (\mathbf{0}, \boldsymbol{\Sigma}_S) \quad \text{and} \quad \mathbf{w}_\psi^c \sim (\mathbf{0}, 2\boldsymbol{\beta}_\psi\,\boldsymbol{\Sigma}_S)\ .$$

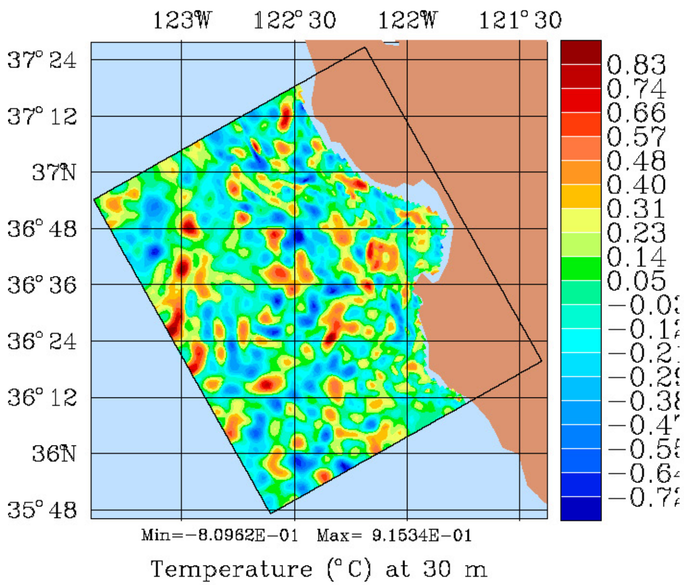
Barotropic Stream Function

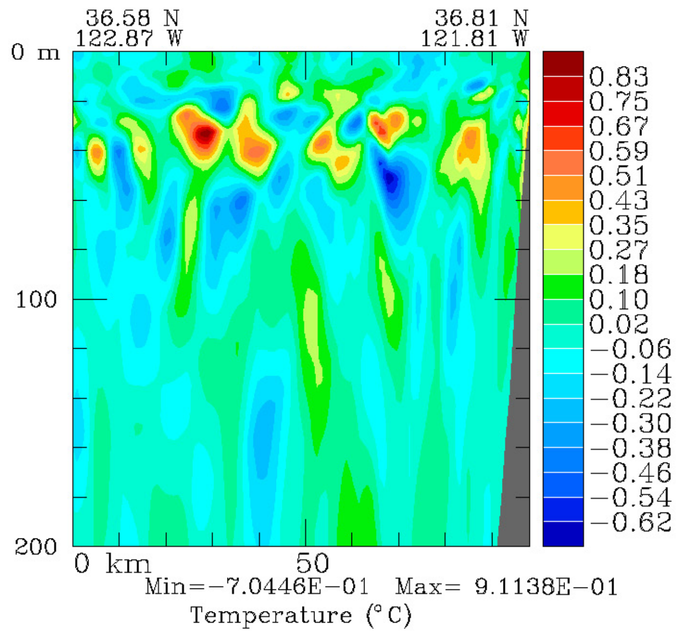
$$\nabla_h \wedge [\mathbf{H}^{-1}\nabla_\mathbf{h} \wedge \mathbf{d}\boldsymbol{\psi}\,\mathbf{e}_3] = -\nabla_h \wedge \mathbf{d\overline{u}} + \mathbf{B}_\psi^{fc}\,\mathbf{d\tilde{w}}_\psi^c\ ,$$

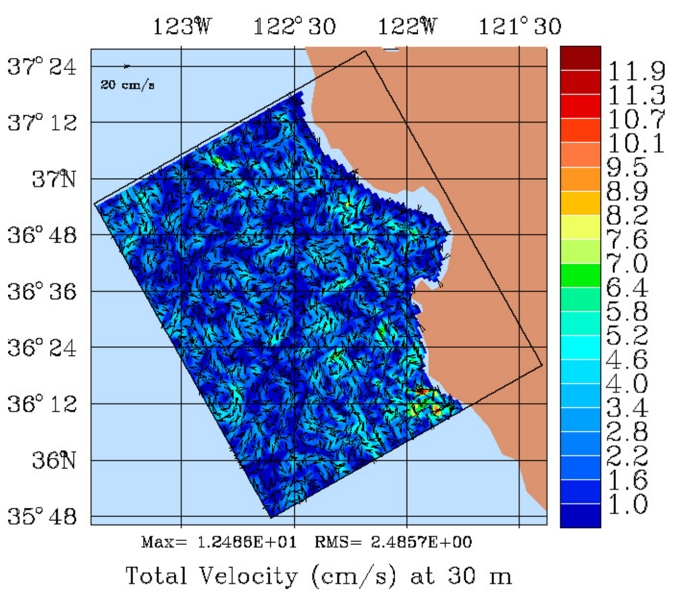
$$\mathbf{d\tilde{w}}_\psi^c = -\boldsymbol{\beta}_\psi\,\tilde{\mathbf{w}}_\psi^c\,dt + \mathbf{dw}_\psi^c\ .$$

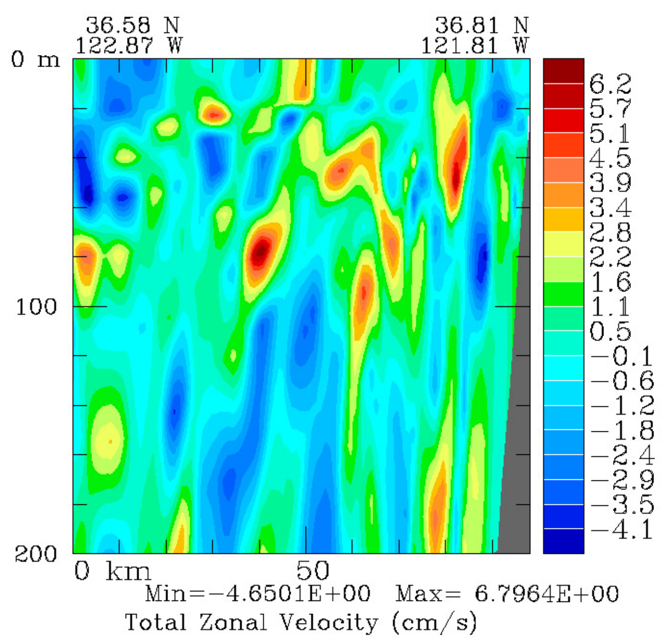
$$\text{with} \quad \tilde{\mathbf{w}}_\psi^c(0) \sim (\mathbf{0}, \boldsymbol{\Sigma}_\psi) \quad \text{and} \quad \mathbf{w}_\psi^c \sim (\mathbf{0}, 2\boldsymbol{\beta}_\psi\,\boldsymbol{\Sigma}_\psi)\ .$$

Effects of Stochastic Forcings







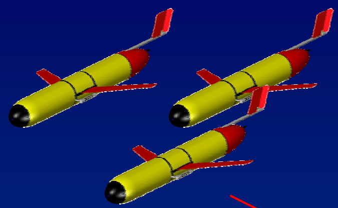


Differences between a deterministic and stochastic PE simulation, after 1-day of integration

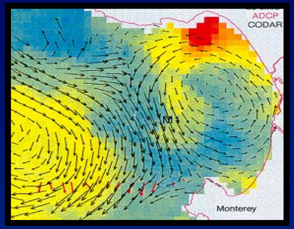
Left: Differences in horizontal maps of *T* (top) and $||u_h||$ (bottom) at 30m depth

Right: Differences in cross-sections (from offshore to the coast in Monterey Bay) of *T* and *u*, from 0 to 200m depth.

Amplitudes of stochastic forcings set to "ε x ||geostrophy(*z*)||", with a 1/2 day decorrelation in time and 1-to-2 grid point correlation in space.

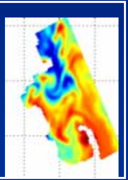


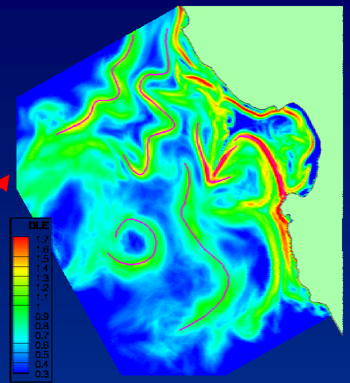
Glider data



HF Radar

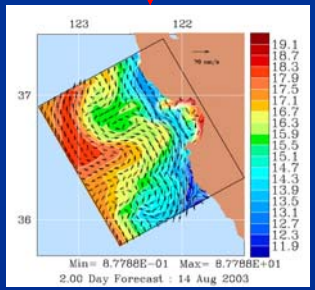
HOPS
ESSE



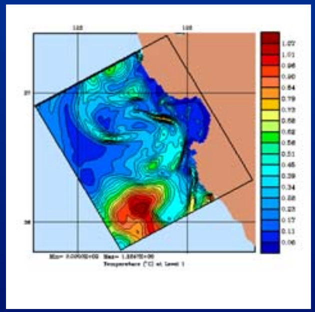


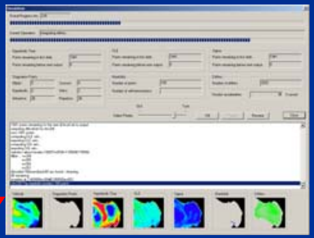
Forecast DLE

Forecast velocities

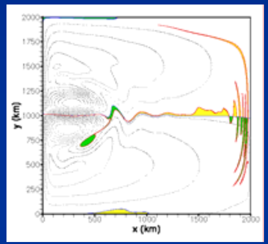


Forecast uncertainty (in ESSE ensemble form)

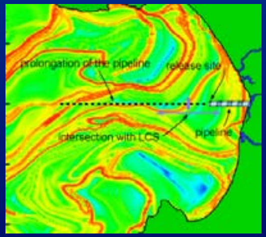


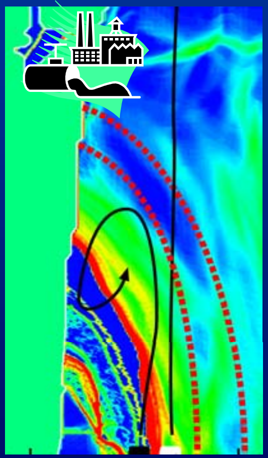


MANGEN



Lobe dynamics
Quantified transport



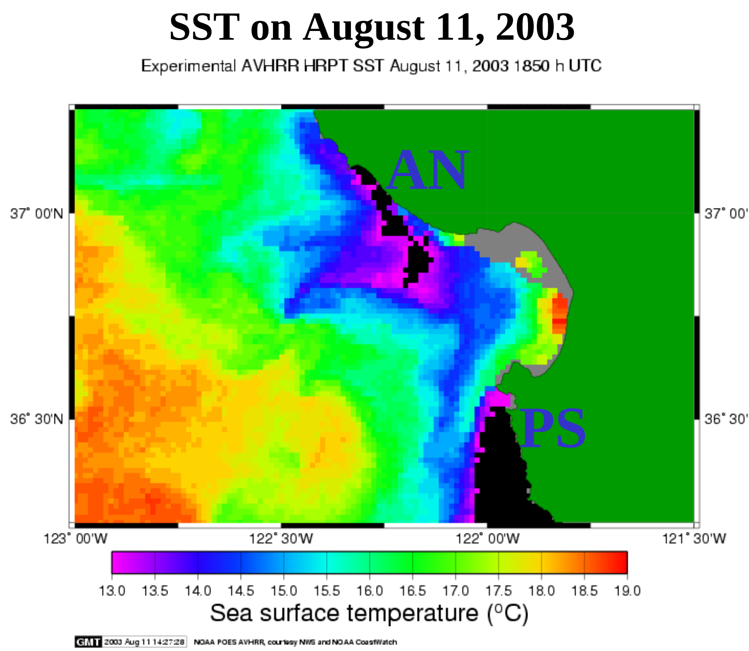


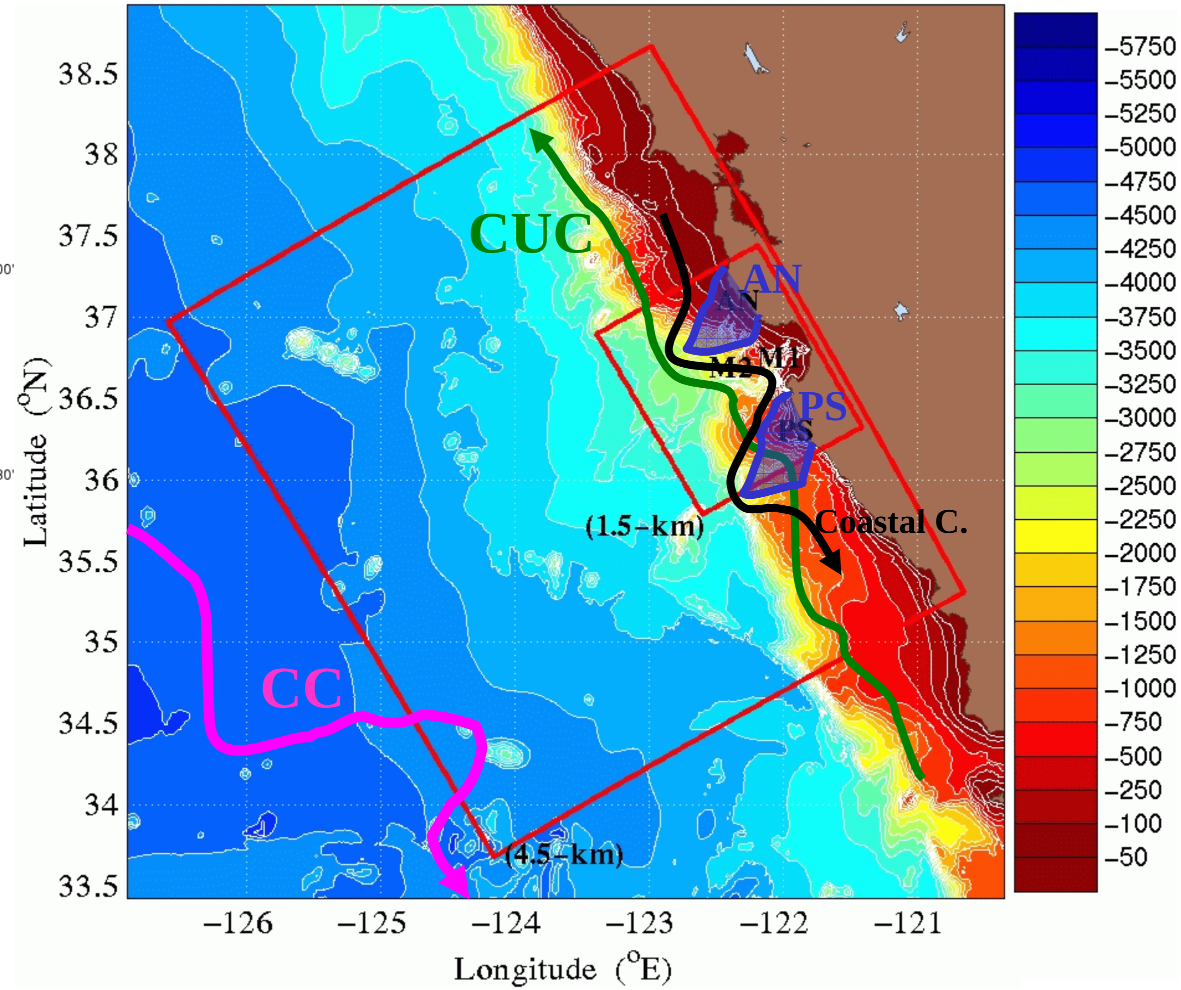
Adaptive Sampling

Efficient navigation

Actuated systems

REGIONAL FEATURES of Monterey Bay and California Current System and Real-time Modeling Domains (AOSN2, 4 Aug. – 3 Sep., 2003)

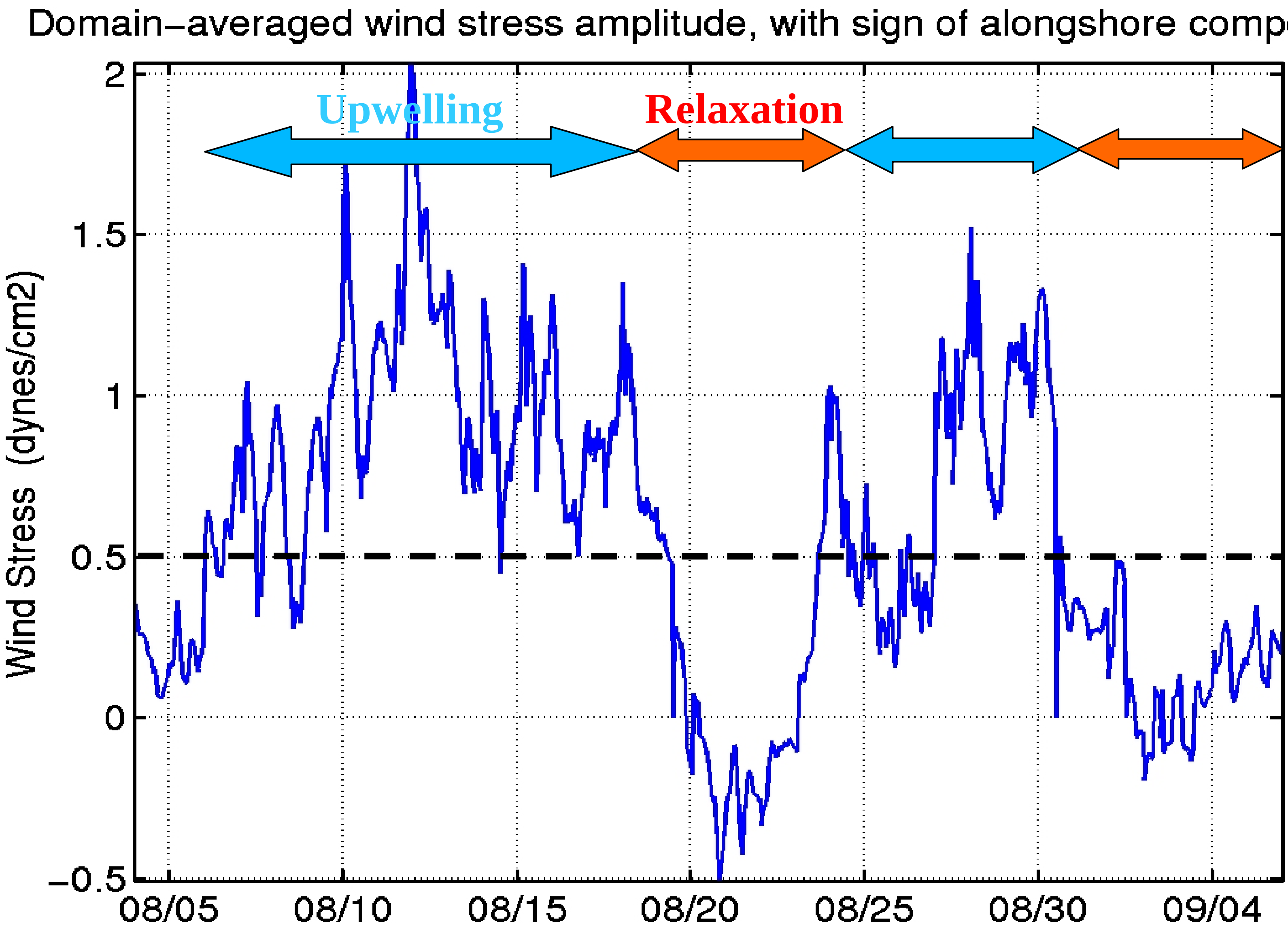




REGIONAL FEATURES

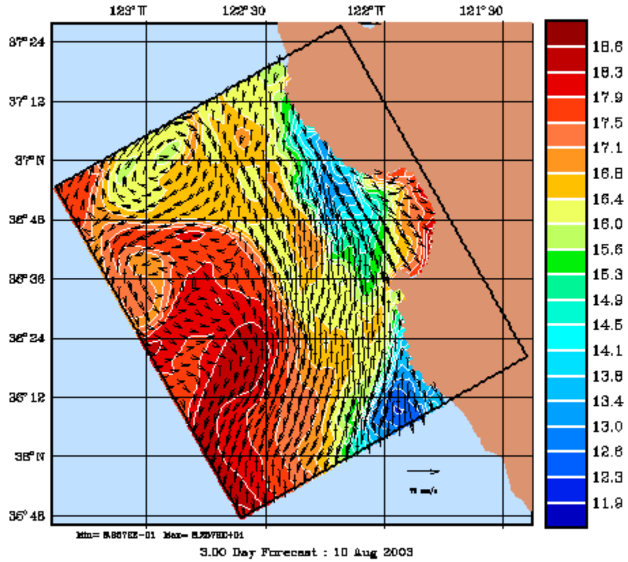
- **Upwelling centers at Pt AN/ Pt Sur:**…………Upwelled water advected equatorward and seaward
- **Coastal current, eddies, squirts, filam., etc:**….Upwelling-induced jets and high (sub)-mesoscale var. in CTZ
- **California Undercurrent (CUC):**……………..Poleward flow/jet, 10-100km offshore, 50-300m depth
- **California Current (CC):**…………………….Broad southward flow, 100-1350km offshore, 0-500m depth

Oceanic responses and atmospheric forcings during August 2003

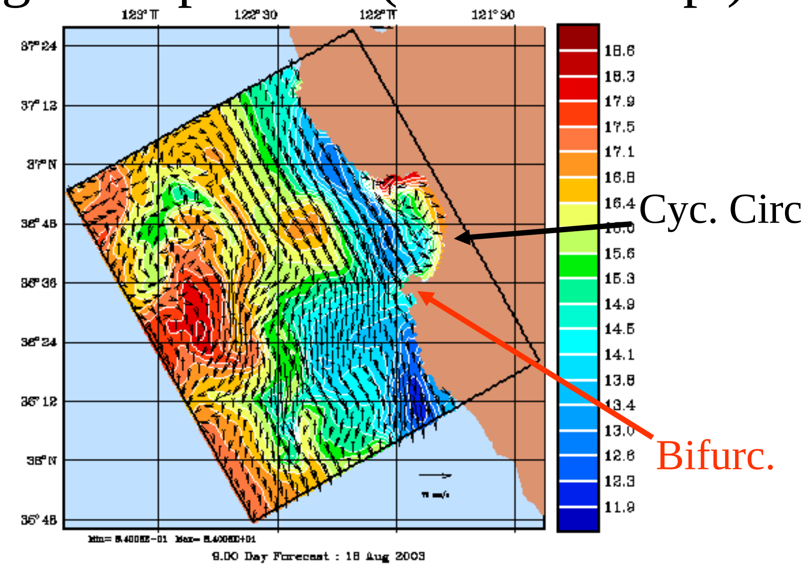


Oceanic responses and atmospheric forcings during August 2003

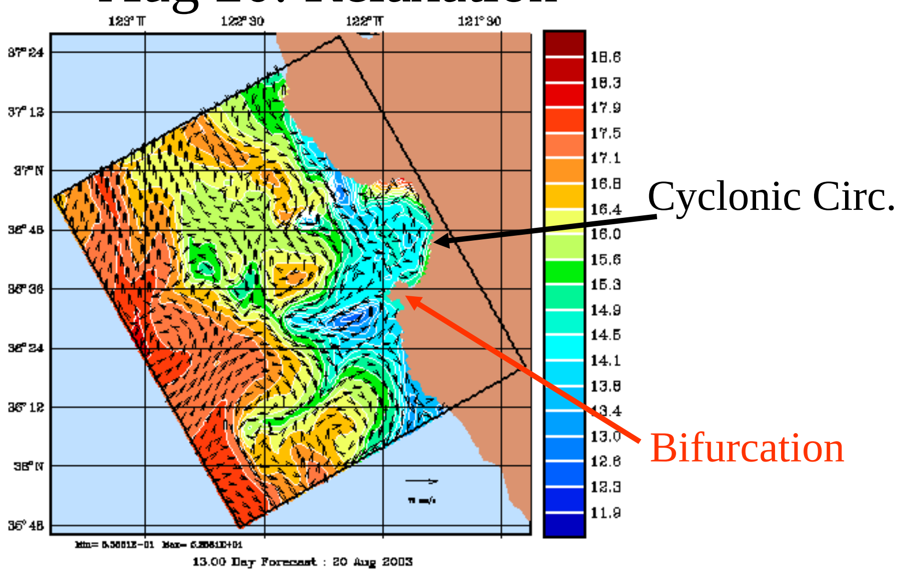
Aug 10: Upwelling (onset)



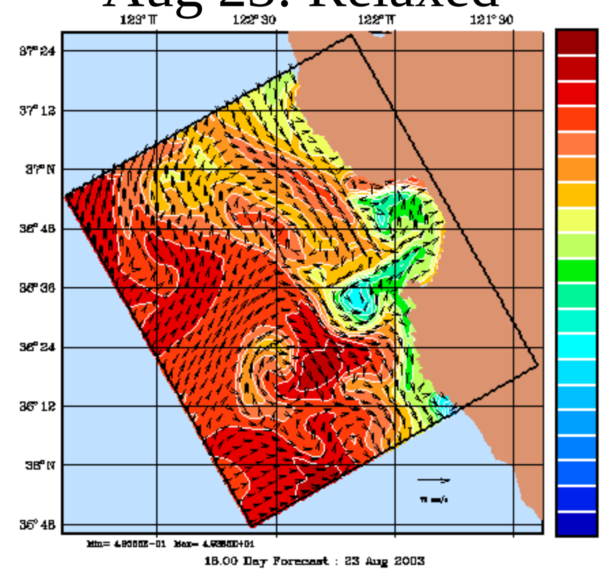
Aug 16: Upwelled (Sustained Up.)



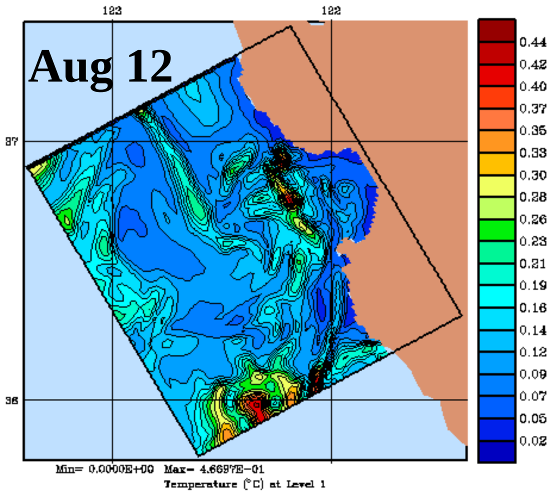
Aug 20: Relaxation



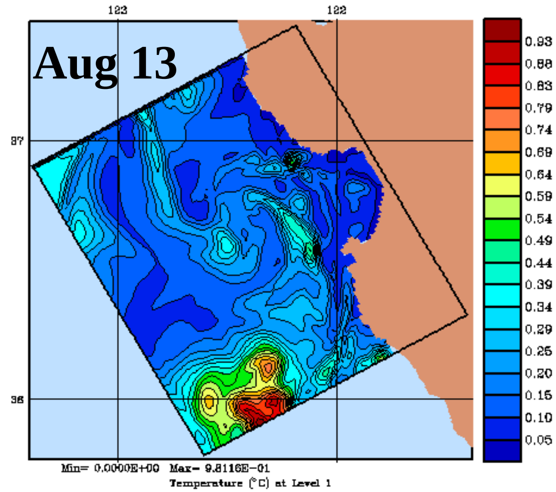
Aug 23: Relaxed

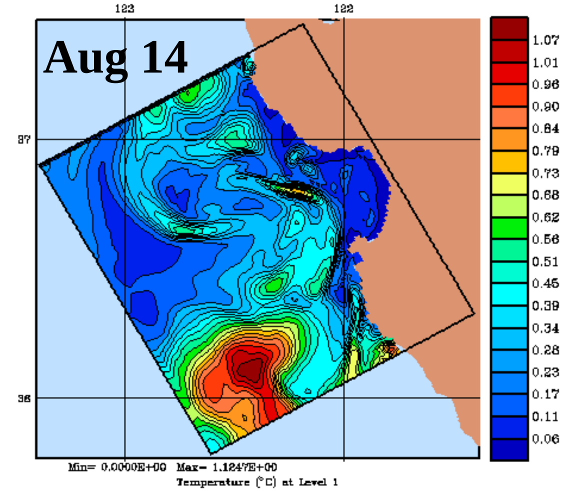


ESSE Surface Temperature Error Standard Deviation Forecasts

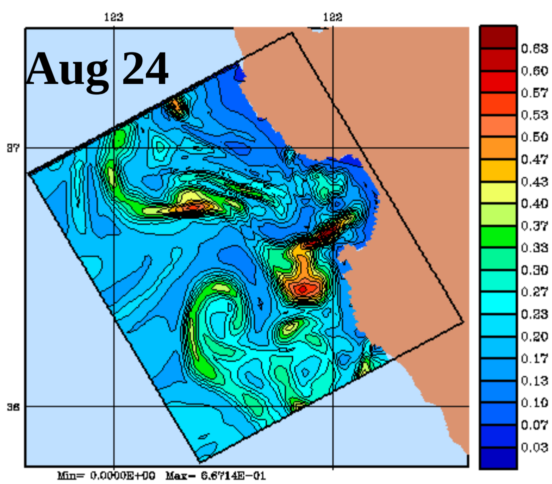


Start of Upwelling

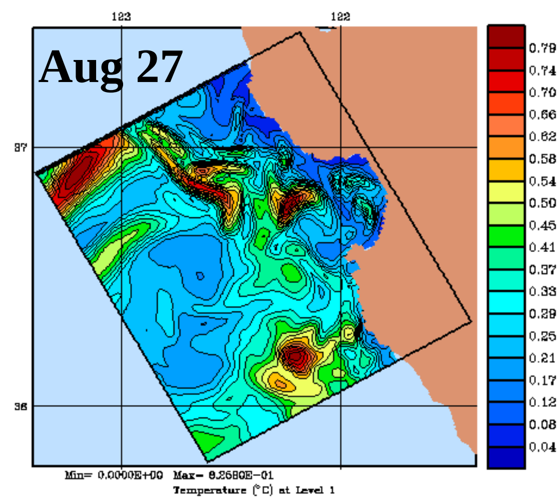


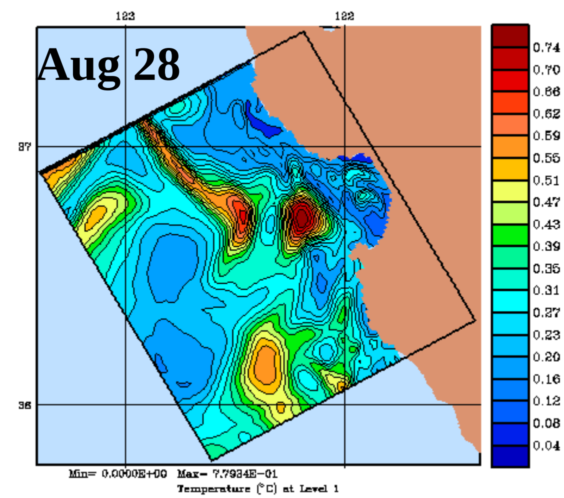


First Upwelling period



End of Relaxation





Second Upwelling period

Uncertainty Estimation Solved here: Prediction problem (hindcast)

1. **Estimate ocean flow field by ensemble ESSE DA approach**
2. **Predict uncertainty for 3-days ahead via ESSE ensemble**
3. **Compute DLE/LCS for each ensemble member (over 3-days)**
4. **Compute Uncertainty on DLE/LCS based on this ensemble**

Note that there are other uncertainty problems:

- Re-analysis (smoothing)
- Predictability limit problem (entropy on/of LCS), etc.

Upwelling: Aug 26 - 29

Predicted Surface Temp. overlaid with velocity vectors

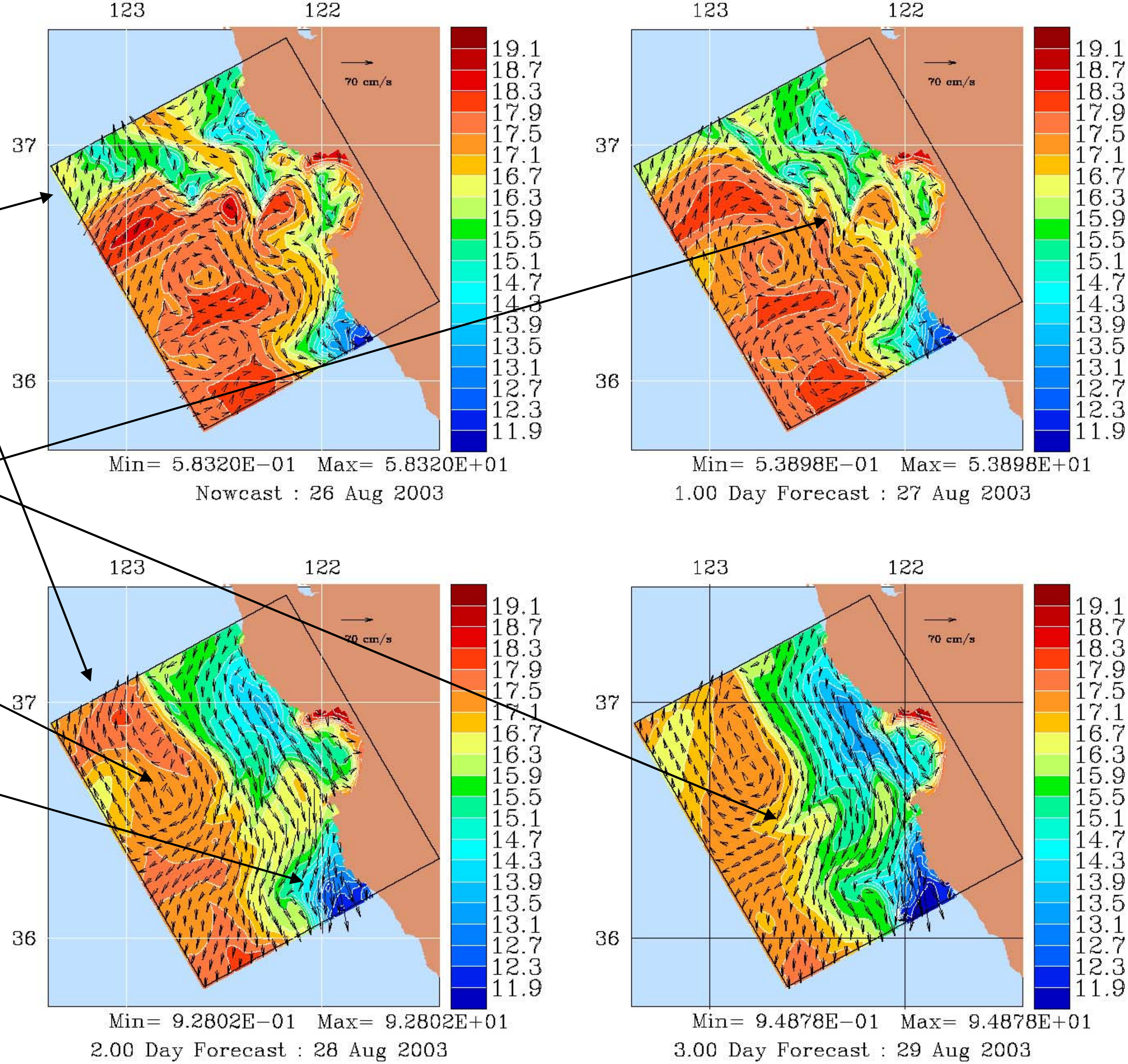
Northward advection of cold eddy field

Pt AN Upwelling Plume/Squirt

Offshore eddy

Pt Sur upwelling and fronts/eddying

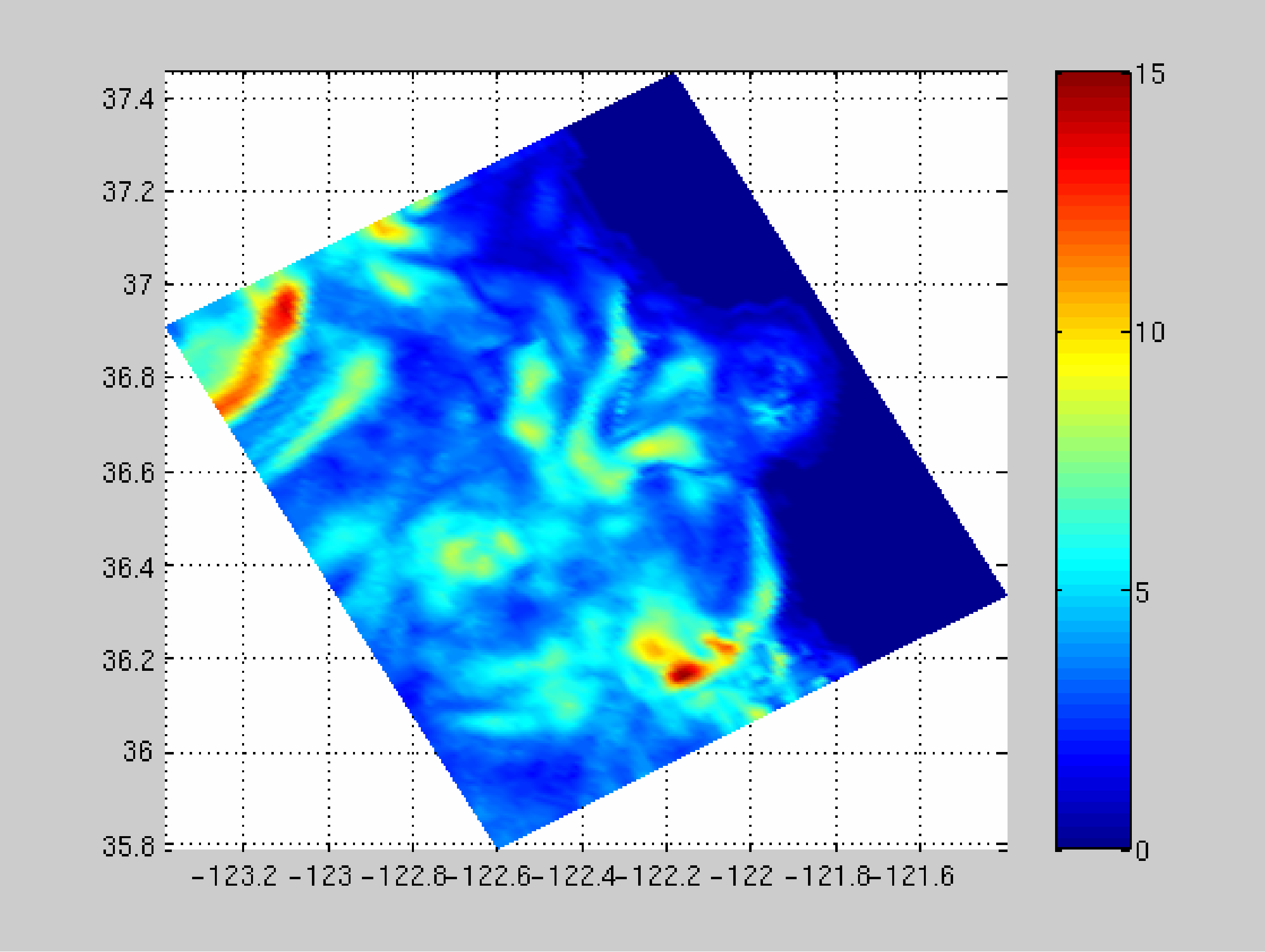
Daily cycles and wind-driven responses



Upwelling (Aug 26-Aug 29): |u| error Std. estimate

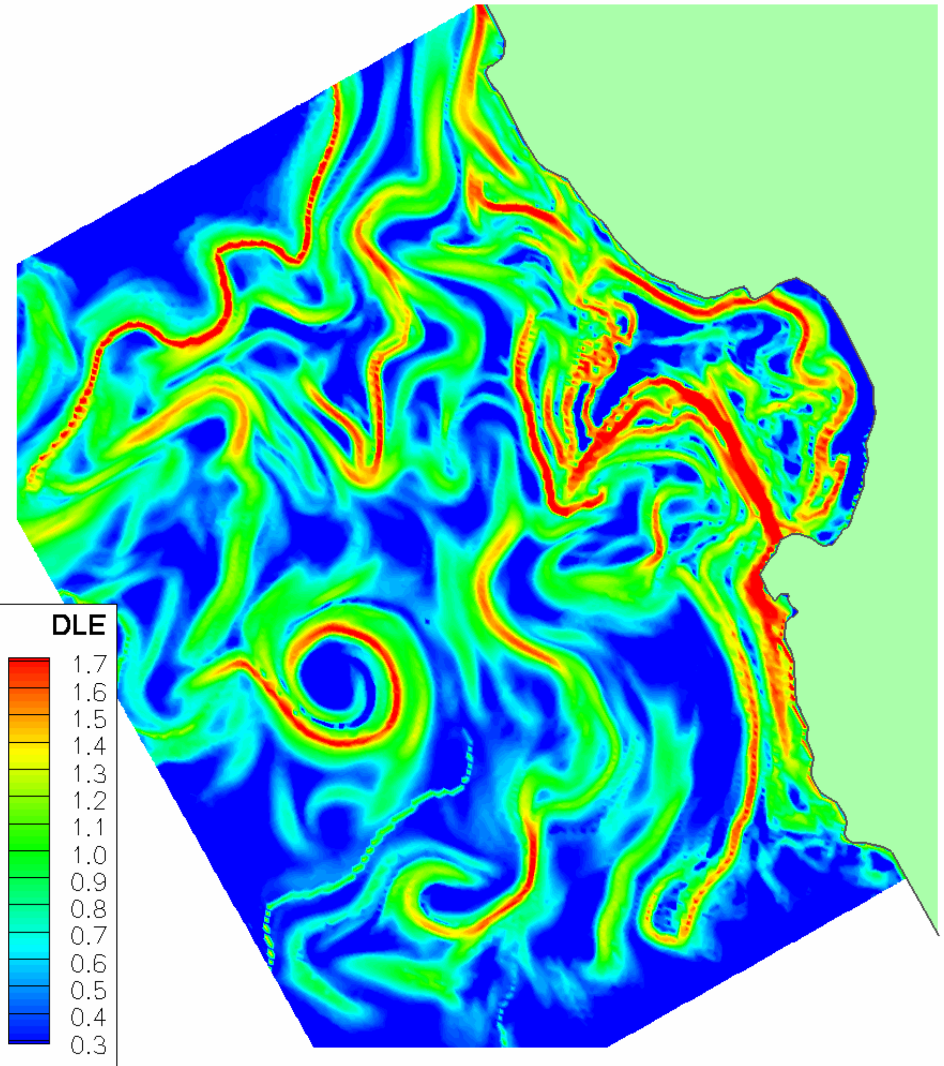
Main features
(from north to south)

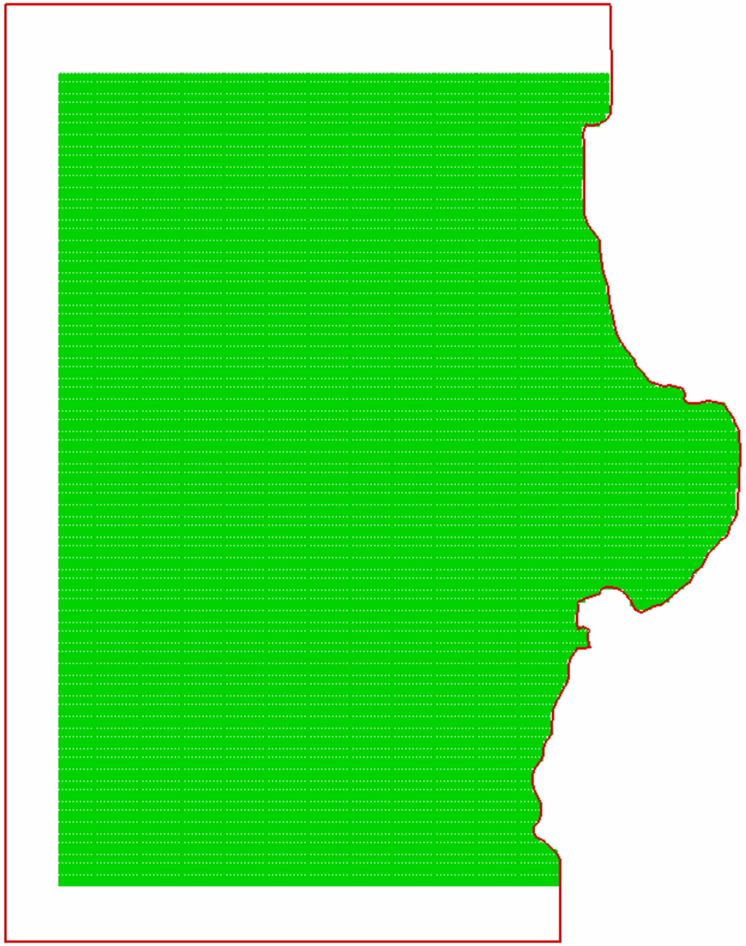
- Uncertainty ICs:
  - -a function of past dynamics and of past measurement types/locations
- Northward advection of cold eddy field
- Pt AN Upwelling Plume/Squirts formation, position and instabilities
- Offshore eddy
- Pt Sur upwelling frontal position and instabilities
- Daily cycles and wind-driven uncertainty reduction/burst



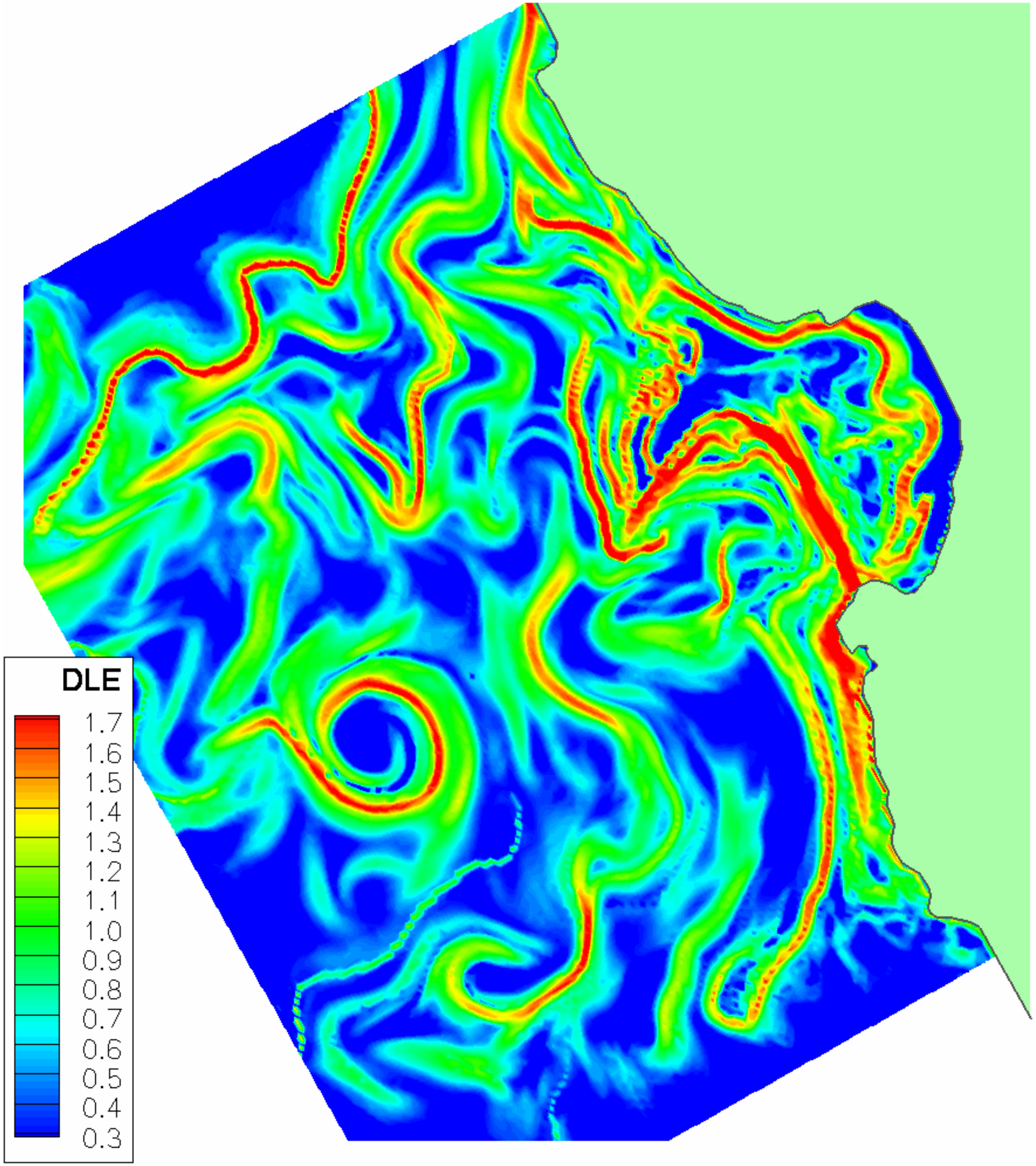
Ocean Realization #1 (hindcast)

Flow field evolution (right) and its DLE for T= 3 days (below)



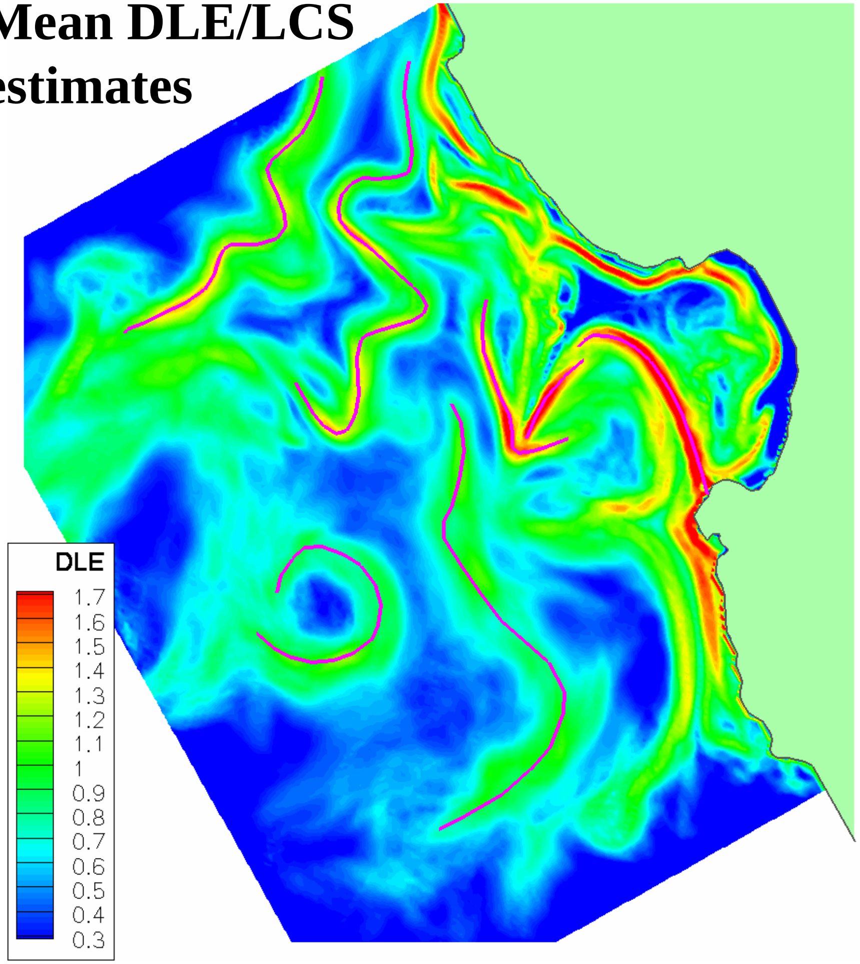


DLE
Realizations #1-26

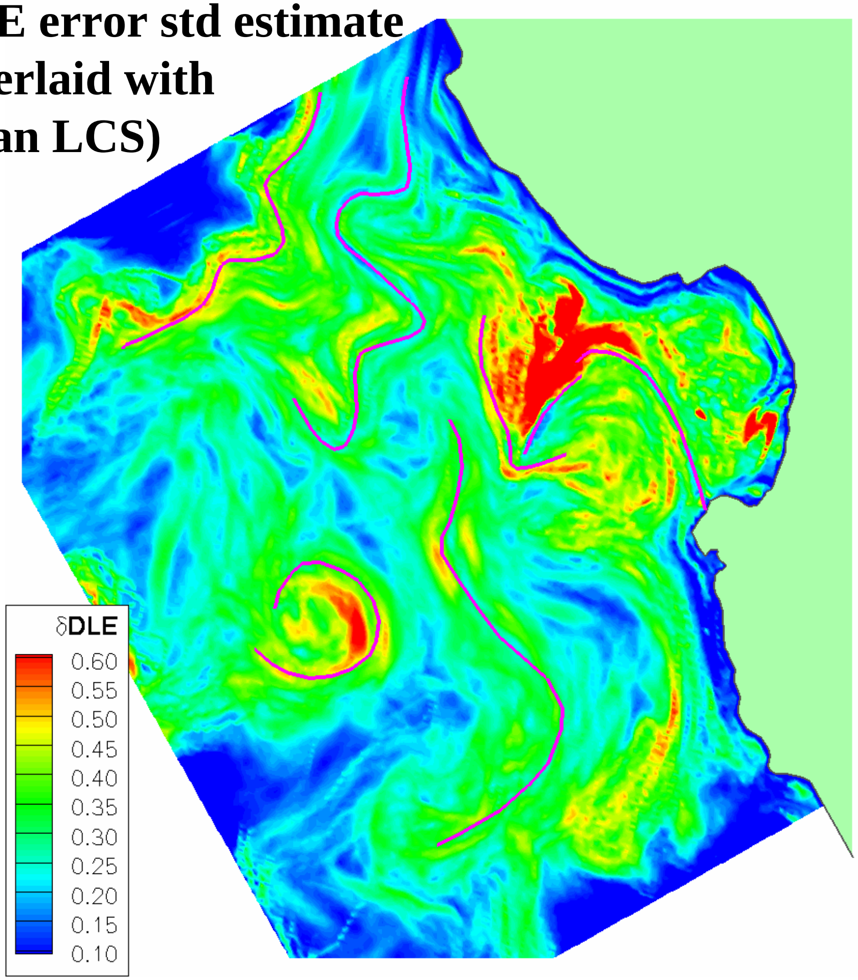


Uncertainties of DLE/LCS

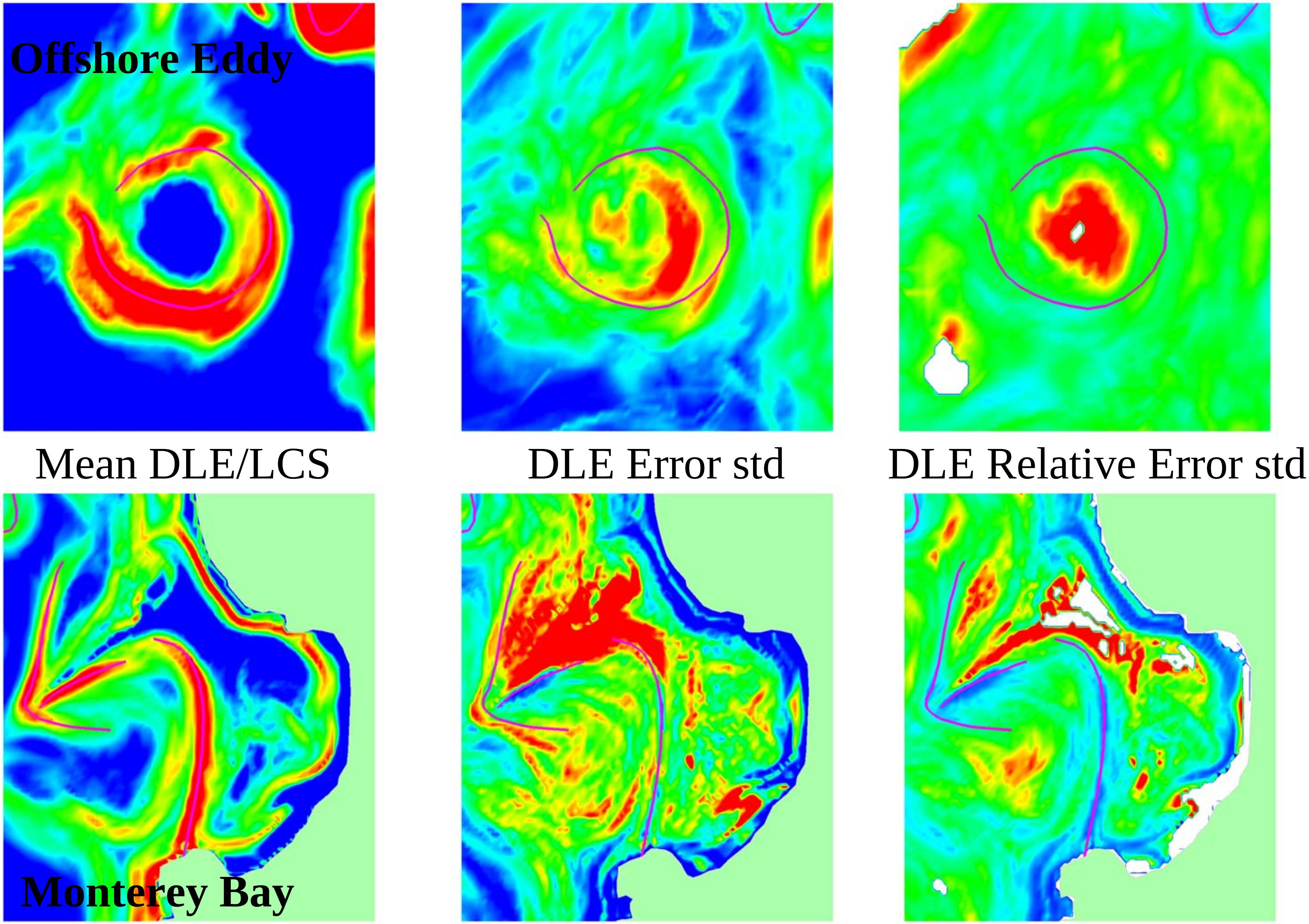
Mean DLE/LCS estimates



DLE error std estimate (overlaid with mean LCS)



Focus on 2 structures: Offshore Eddy and Monterey Bay

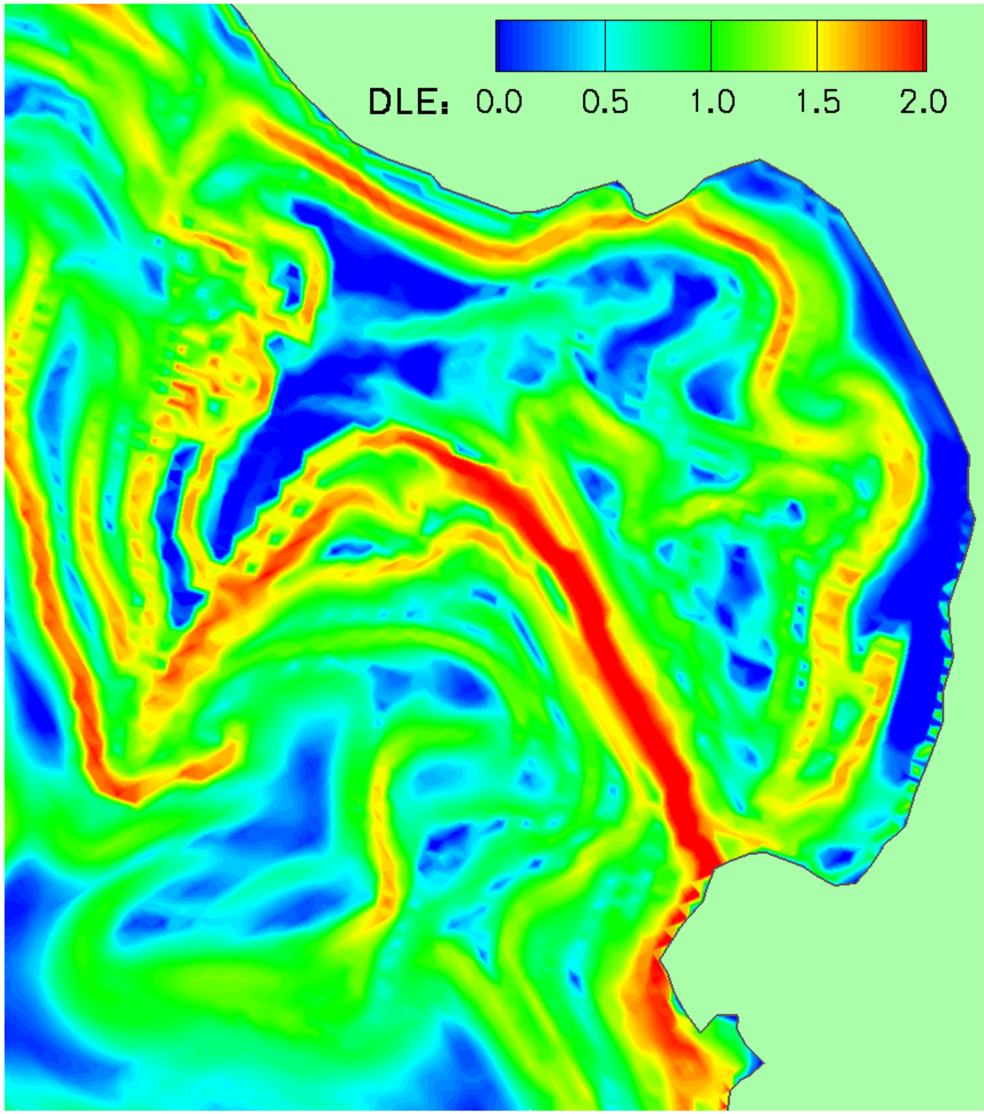


DLE

Realizations #1-26

Zoom over

Monterey Bay region



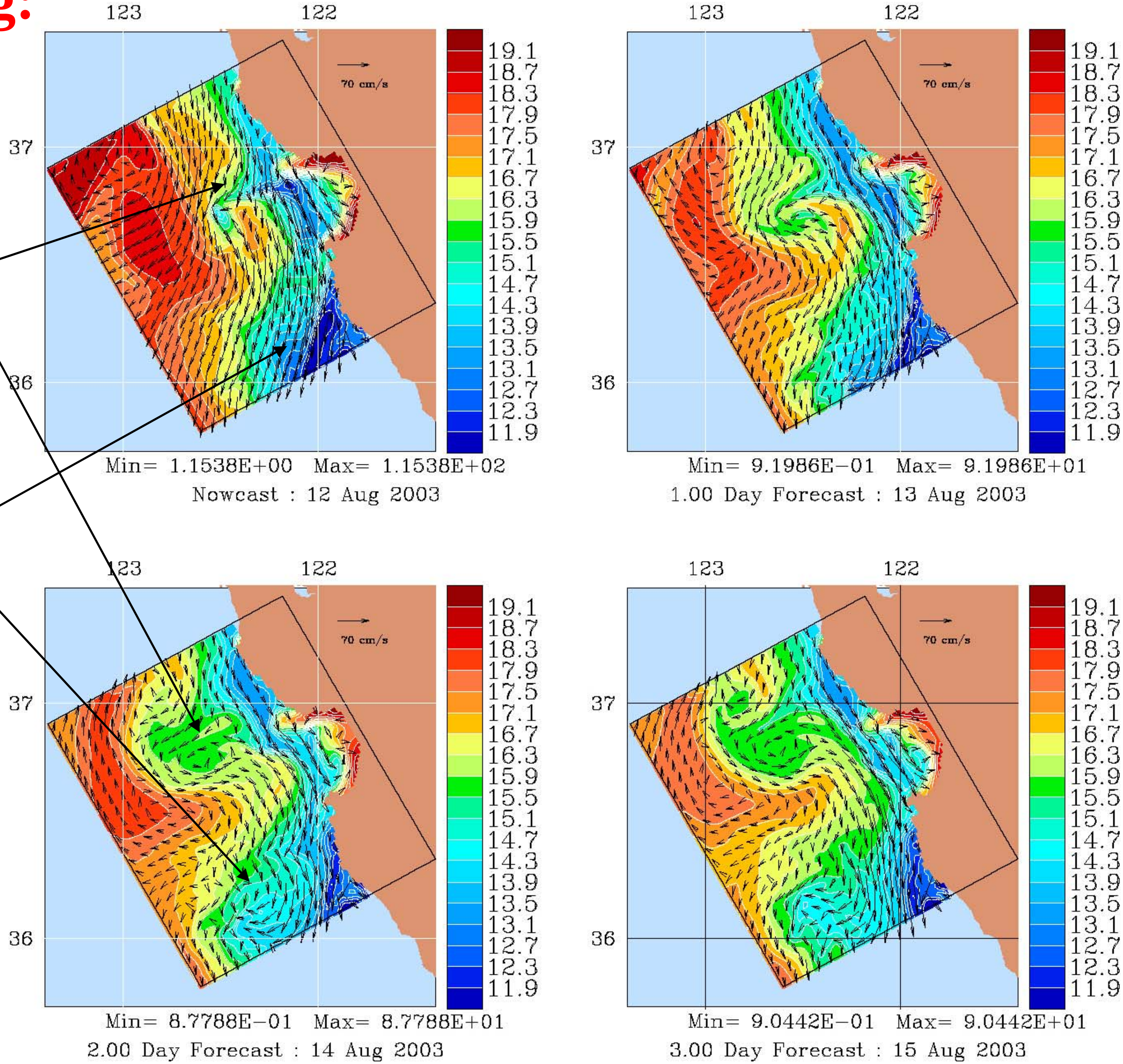
1st Upwelling: Aug 12 - 15

Predicted Surface Temp. overlaid with velocity vectors

Pt AN Upwelling Plume/Squirt and eddying

Pt Sur upwelling and fronts/eddying

Strongly wind-driven regime of surface currents

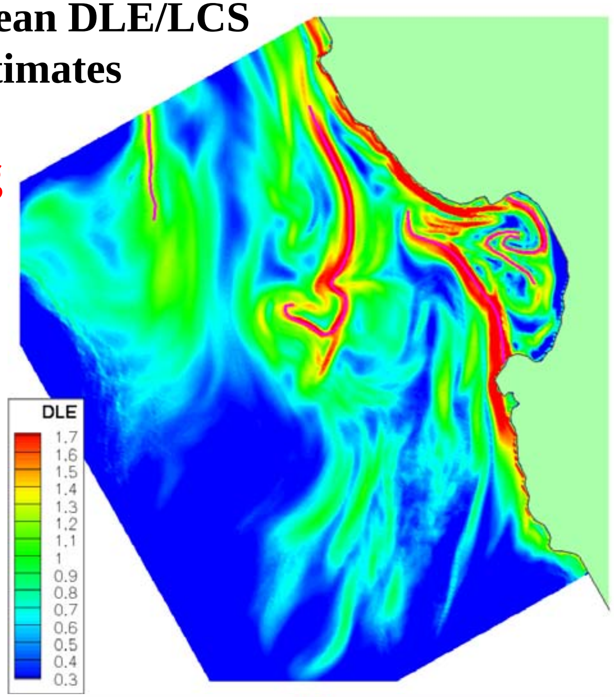


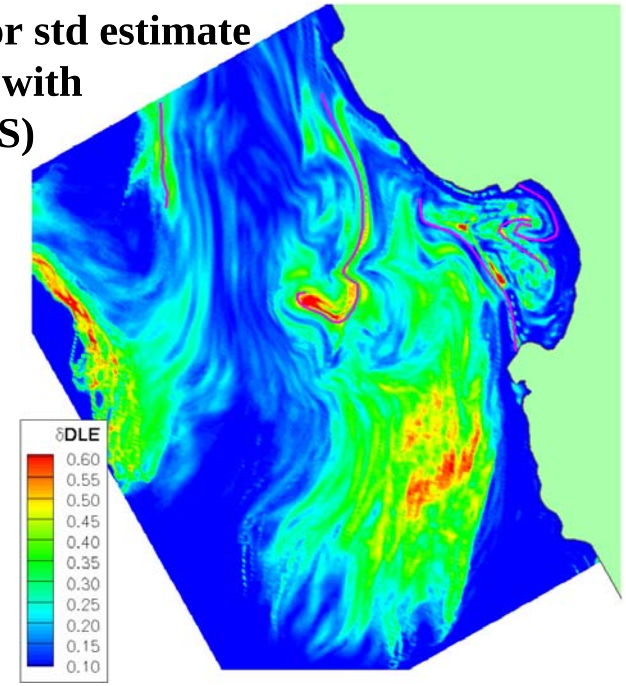
Mean DLE/LCS estimates

DLE error std estimate (overlaid with mean LCS)

1st Upwelling
Aug 7-18

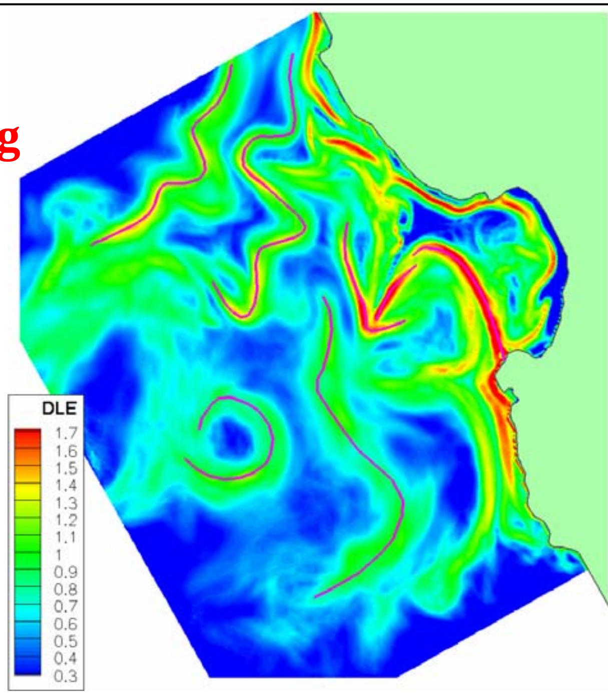
Our 3 days (Aug 12-15) are within Sustained Up. period

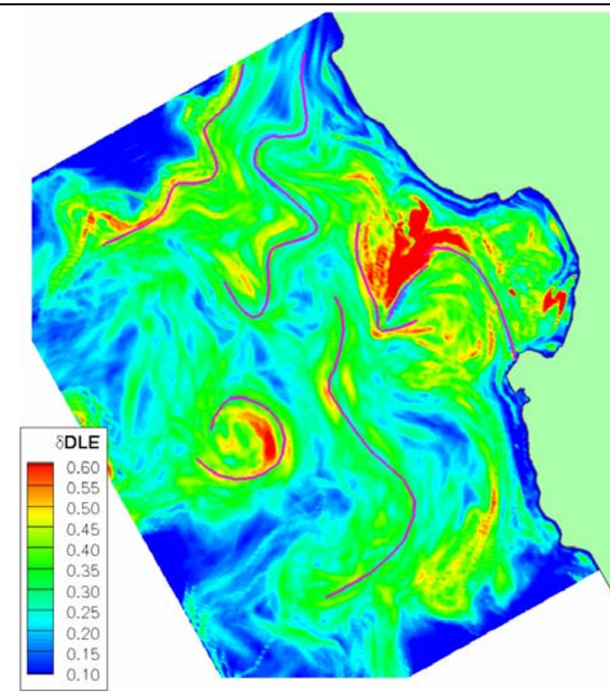




2nd Upwelling
Aug 26-30

Our 3 days (Aug 26-29) are within Up. formation period



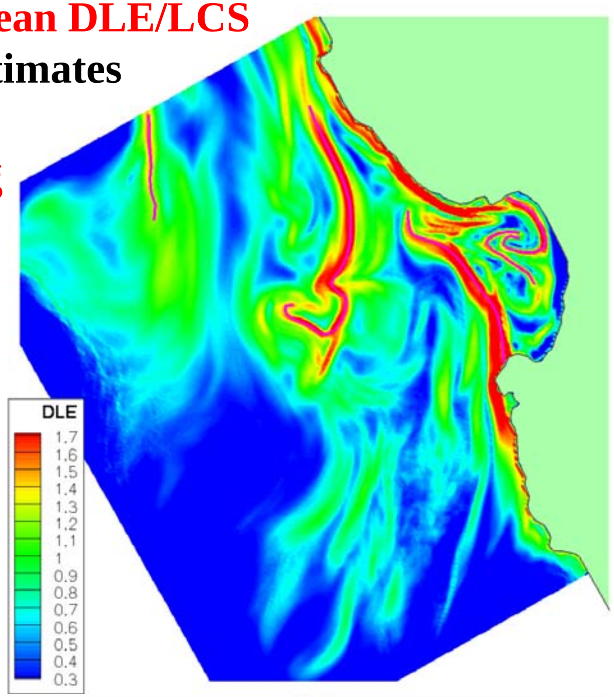


Mean DLE/LCS estimates

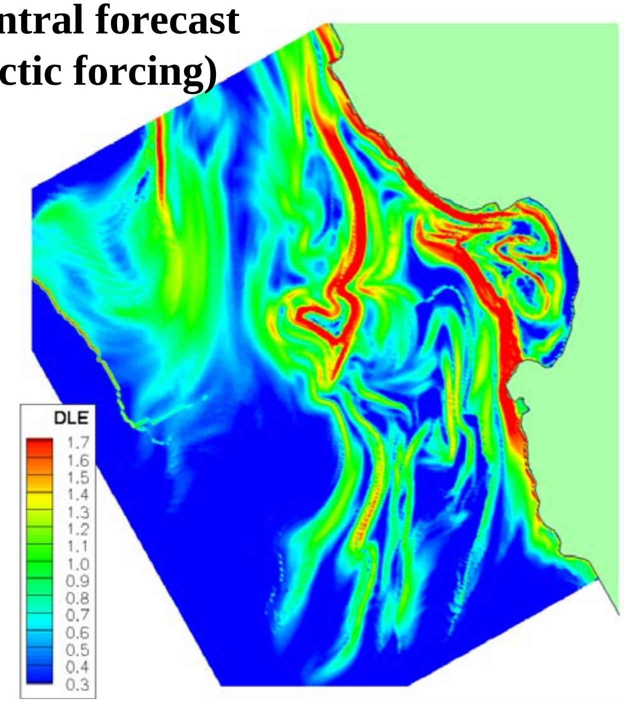
DLE of central forecast (no stochactic forcing)

1st Upwelling Aug 7-18

Our 3 days (Aug 12-15) are within Sustained Up. period

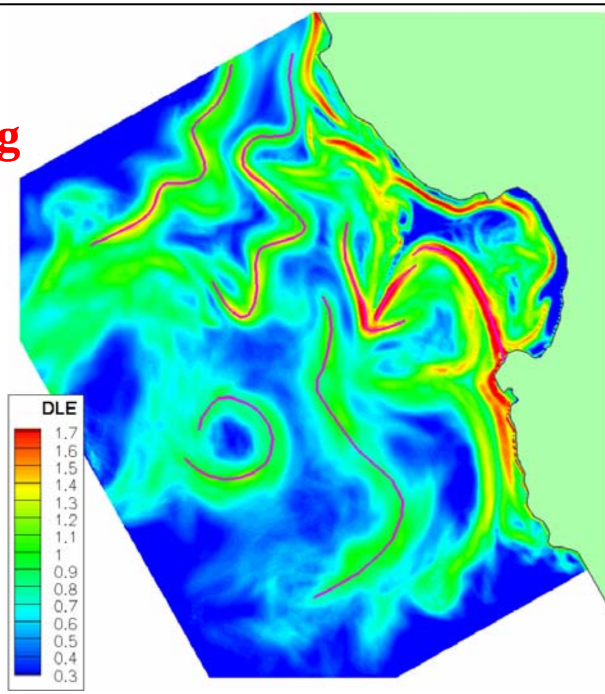


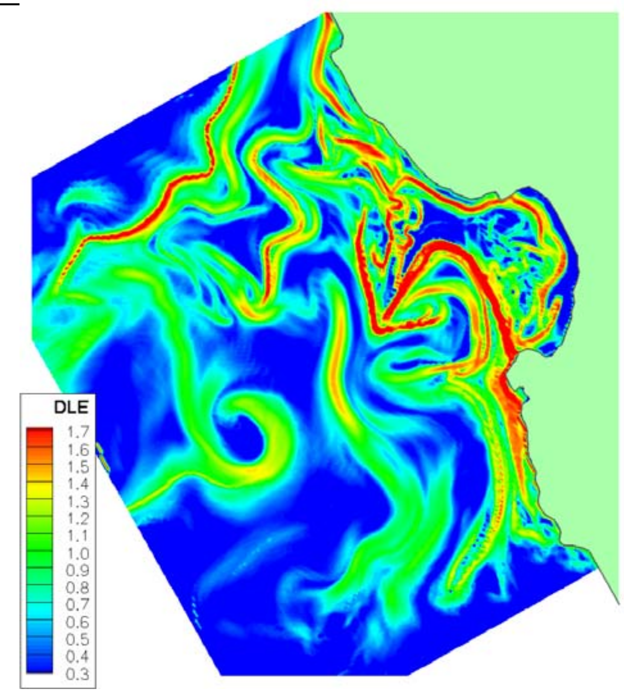
Proxi for DLE of the mean



2nd Upwelling Aug 26-30

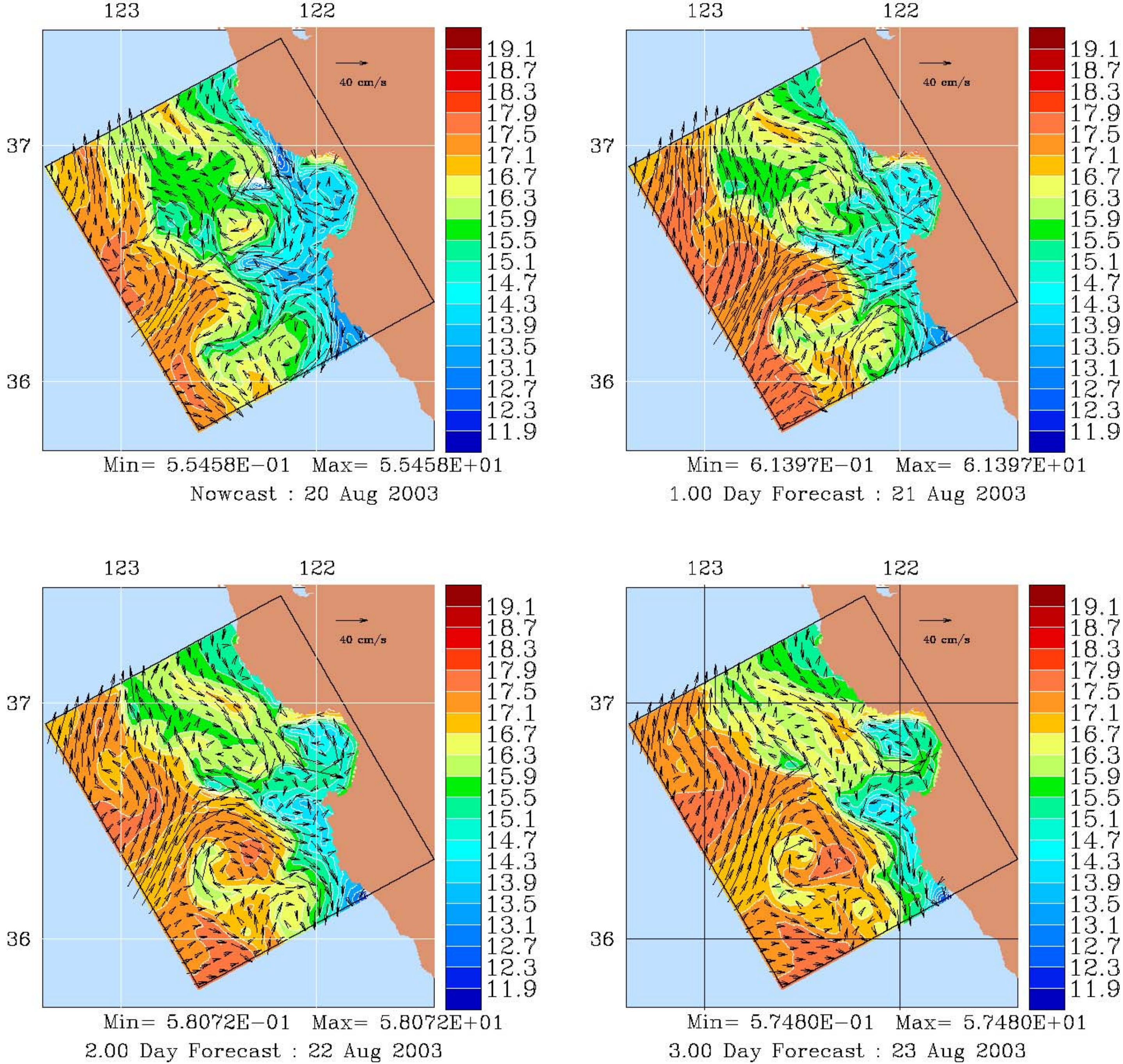
Our 3 days (Aug 26-29) are within Up. formation period



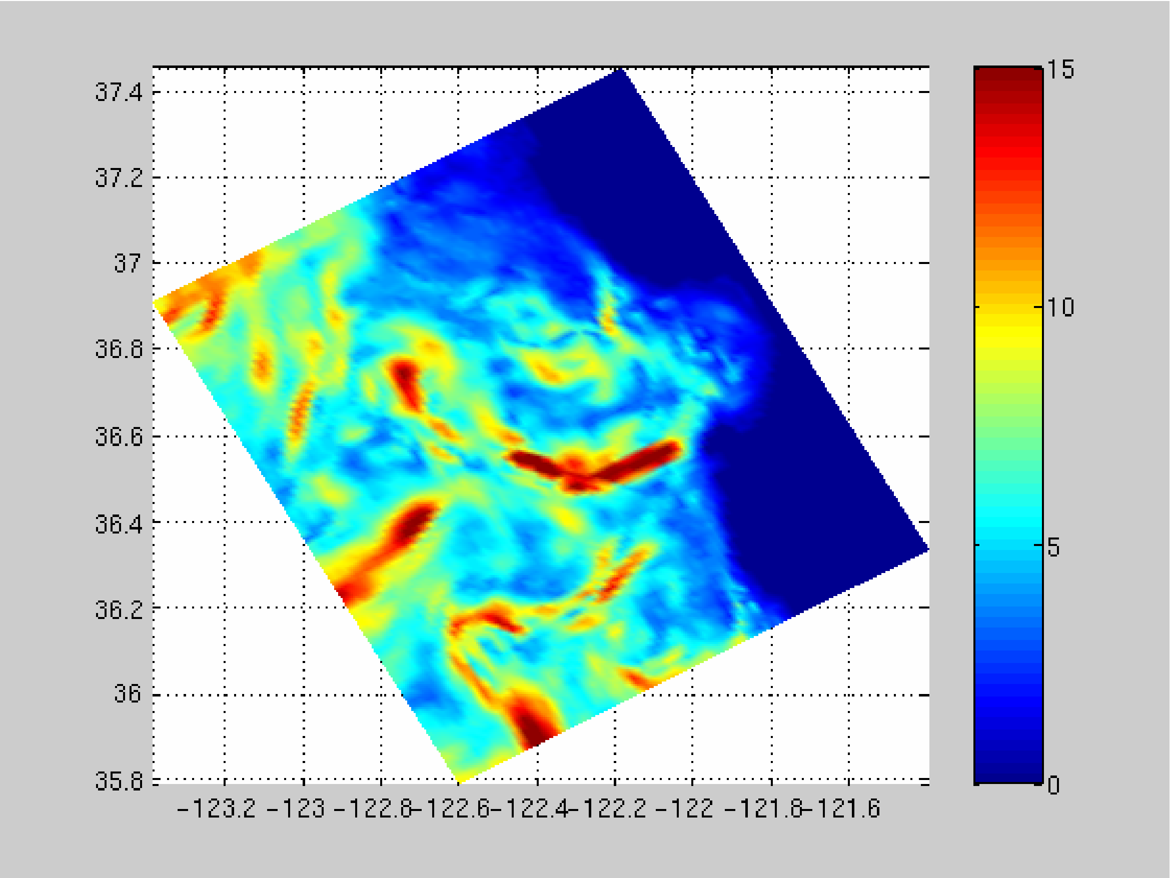


Relaxation
Aug 19-25

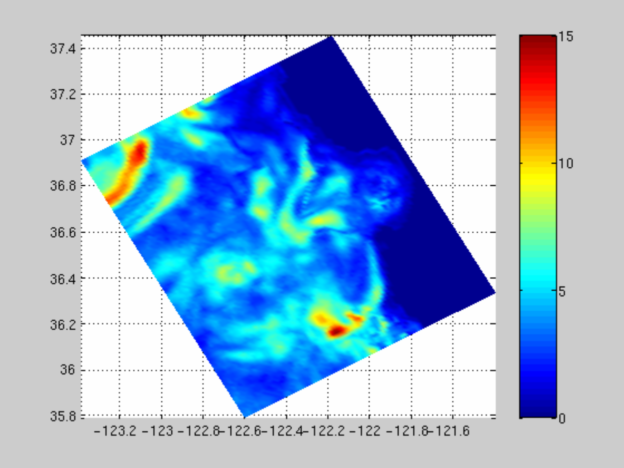
- •Wind-forcing subsides
- •Larger-scale atmos. control eliminated
- •Transfer of kinetic energy to internal ocean dynamics
- •Mesoscale and sub-mesocale instabilities and eddies



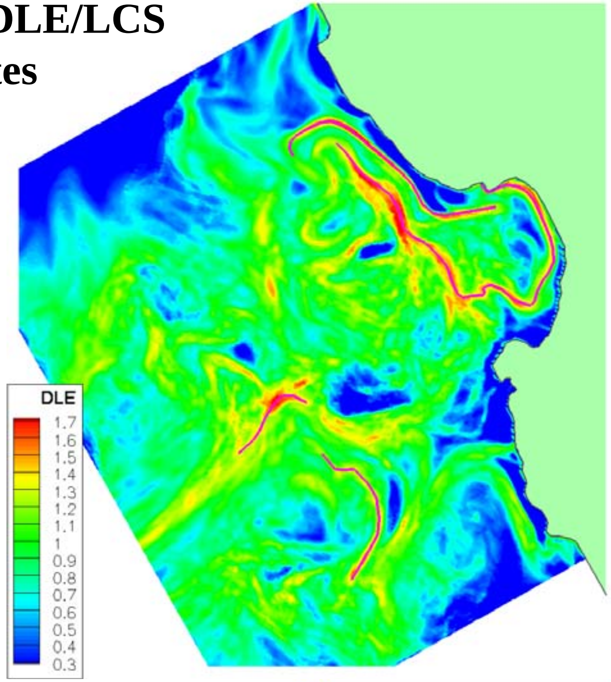
Relaxation (Aug 20-Aug 23): |u| error Std. estimate



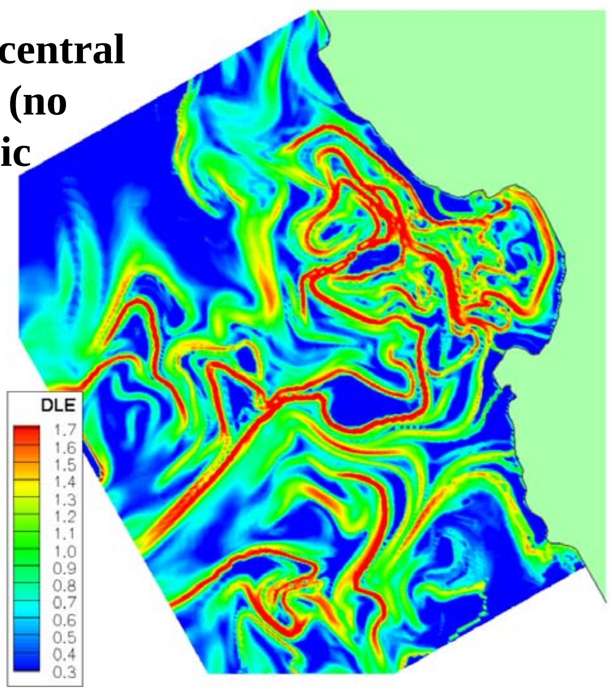
Reminder: Upwelling



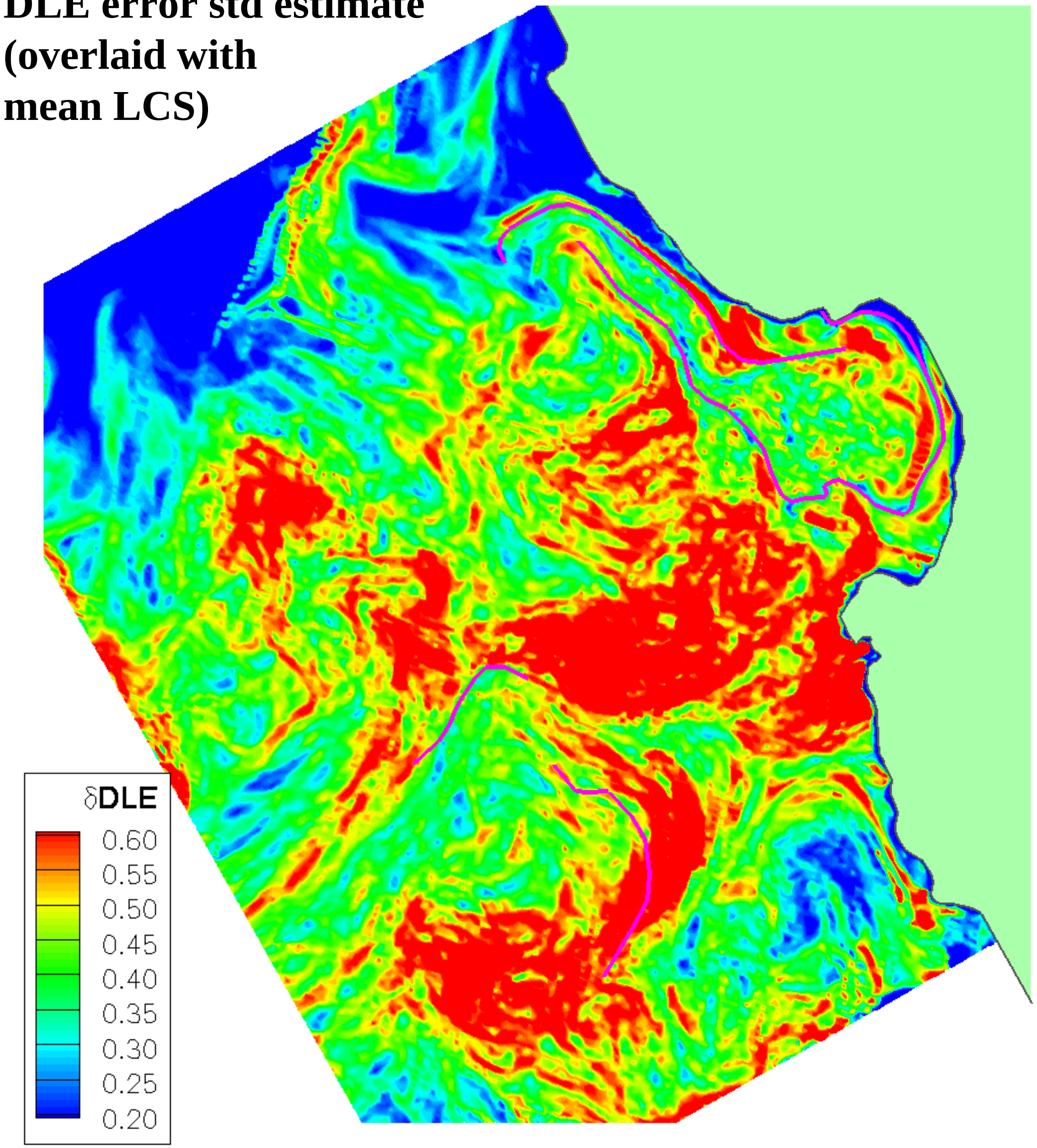
Mean DLE/LCS estimates



DLE of central forecast (no stochastic forcing)



DLE error std estimate (overlaid with mean LCS)



CONCLUSIONS

- HOPS/ESSE and MANGEN combined for useful nonlinear scheme for interdisciplinary estimation of oceanic LCSs and their uncertainties via multivariate ocean data assimilation

- Uncertainty from ensemble of ocean field estimates transferred to ensemble of LCS estimates for 3 prediction problems in the Monterey Bay region: 2 upwelling events and 1 relaxation event

- **Main results:**
  - More intense DLE ridges are usually relatively more certain
  - Different oceanic regimes have different LCS uncertainty properties
  - Strong atmospheric forcing organizes the ocean flow and favors a small number of stronger surface LCS's and uncertainty localization
  - Relaxed atmospheric forcing allows smaller-scale oceanic features to develop/interact, leading to more homogeneous and uncertain LCS's.

- **Work in progress:**
  - Statistics of LCS uncertainties (convergence, pdfs, higher moments)
  - Studies in LCS space: pdf along LCS arc, assign pdf to each LCS, etc
  - Physical-Biological LCS and uncertainties
  - LCS data assimilation

DLE error std estimate (overlaid with mean LCS) for 3 dynamical events