The transfer of uncertainties through physical-acoustical-sonar/signal processing end-to-end systems: a conceptual basis

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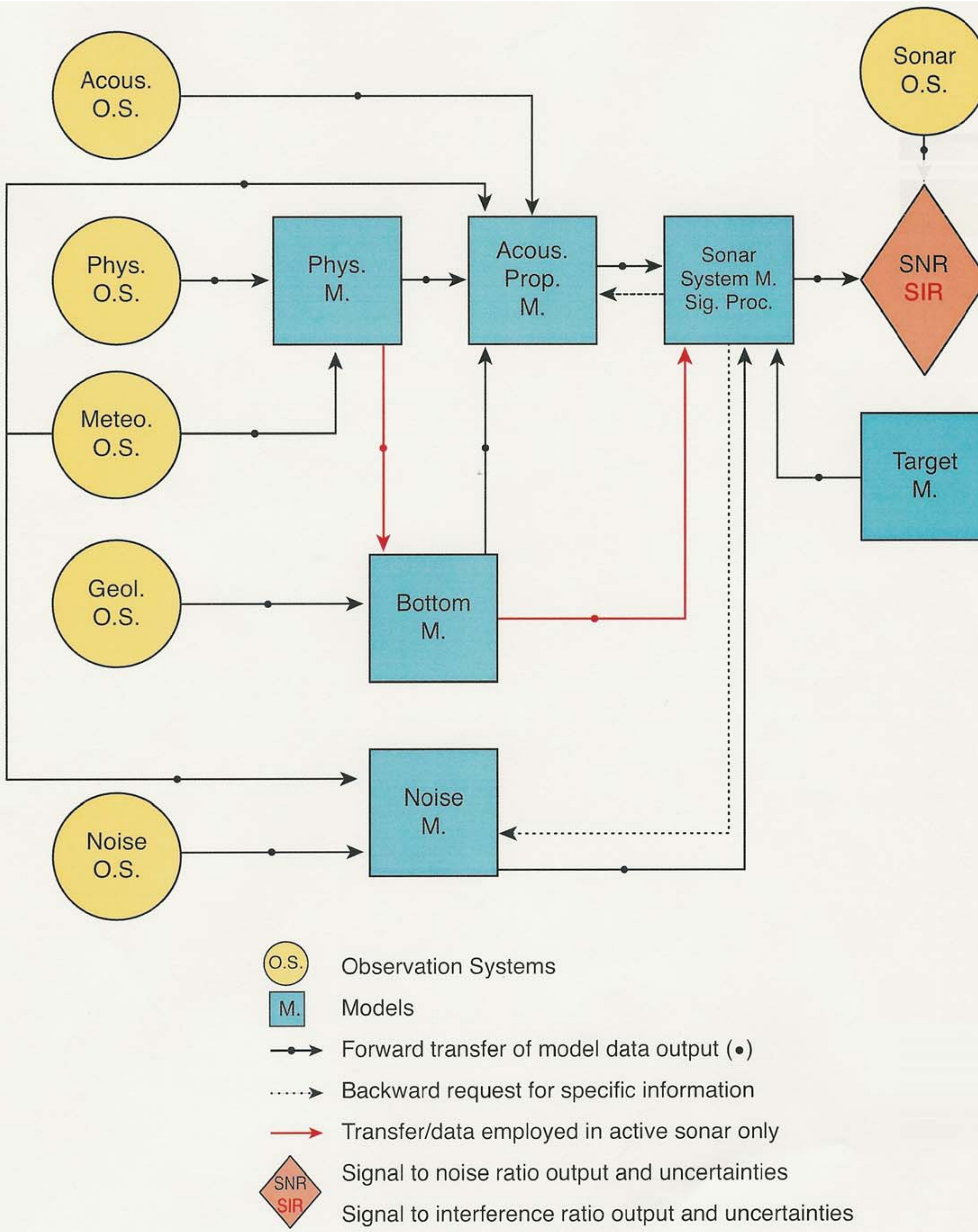
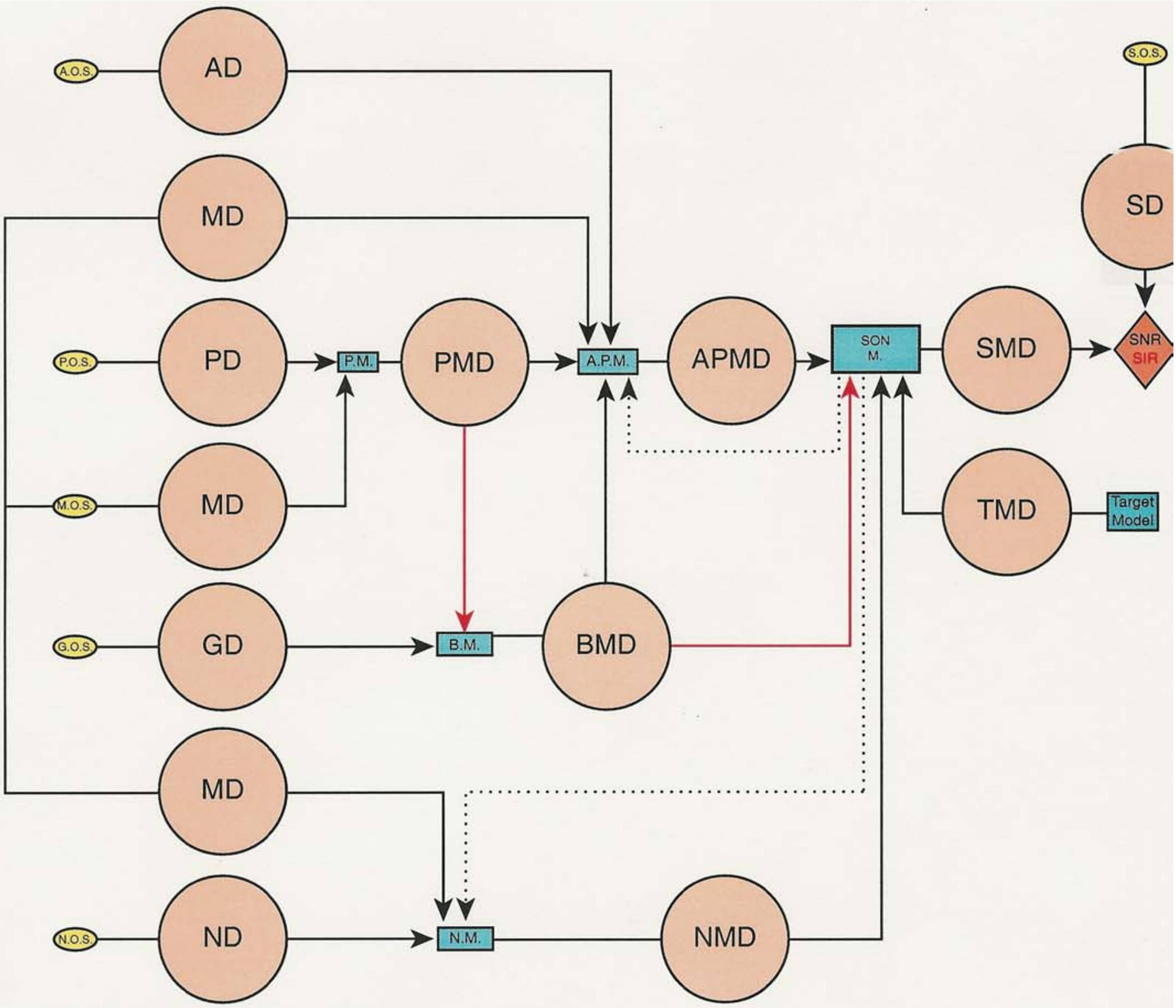
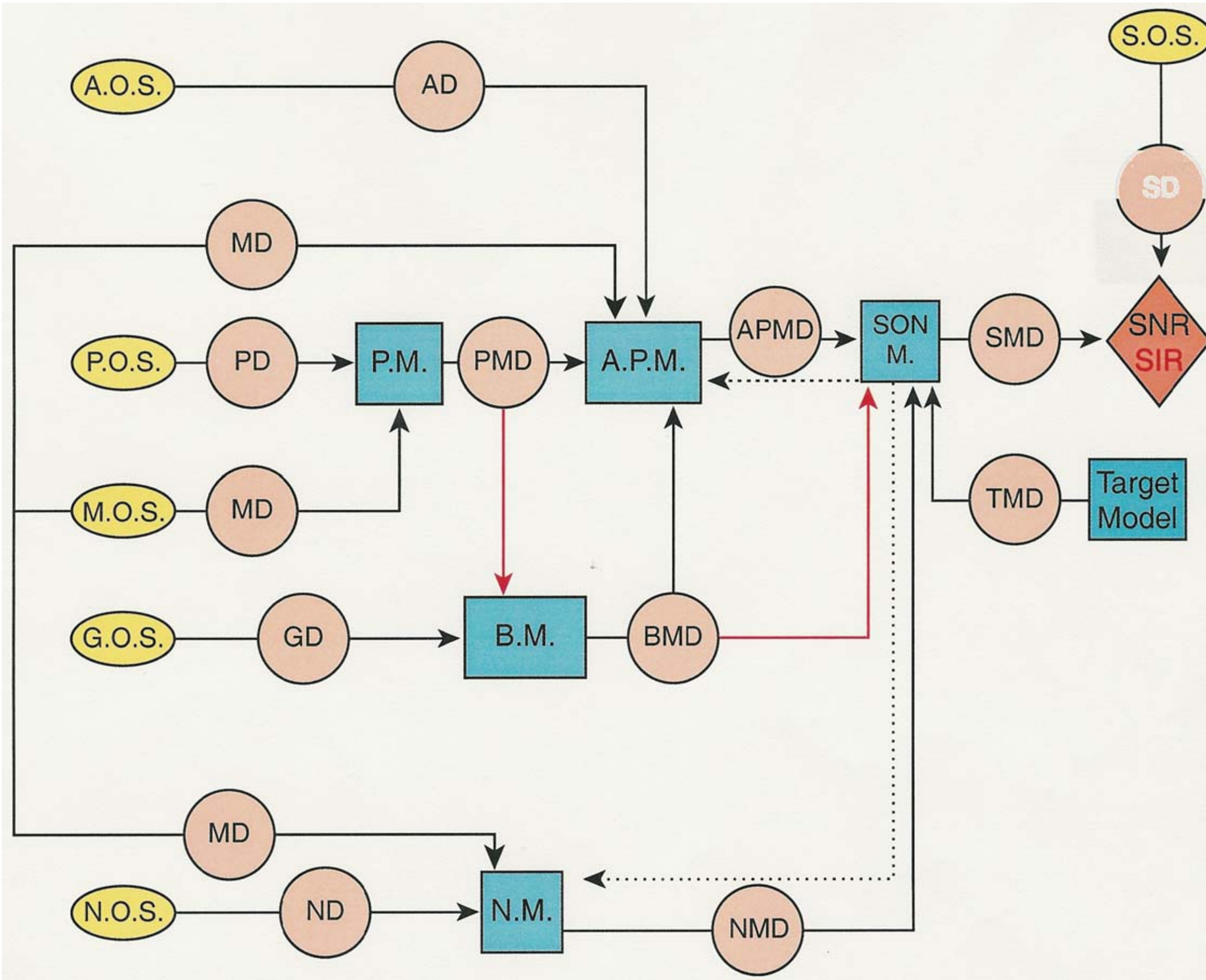

Acous.
O.S.
Sonar
O.S.
Phys.
O.S.
Phys.
M.
Acous.
Prop.
M.
Sonar
System M.
Sig. Proc.
SNR
SIR
Meteo.
O.S.
Target
M.
Geol.
O.S.
Bottom
M.
Noise
M.
Noise
O.S.
O.S.
Observation Systems
M.
Models
Forward transfer of model data output (•)
Backward request for specific information
Transfer/data employed in active sonar only
SNR
SIR
Signal to noise ratio output and uncertainties
Signal to interference ratio output and uncertainties



AD: Acoustical Data
MD: Meteorological Data
PD: Physical Data
GD: Geological Data
ND: Noise Data
SD: Sonar Data
PMD: Physical Model Data
BMD: Bottom Model Data
NMD: Noise Model Data
APMD: Acoustic Propagation Model Data
SMD: Sonar Model Data
TMD: Target Model Data



S.O.S.
A.O.S.
AD
SD
MD
SNR
SIR
APMD
SON
M.
SMD
P.O.S.
PD
P.M.
PMD
A.P.M.
M.O.S.
MD
TMD
Target
Model
G.O.S.
GD
B.M.
BMD
MD
N.O.S.
ND
N.M.
NMD

COUPLED PHYSICAL-ACOUSTICAL DYNAMICAL MODELS

- **Physical model: Primitive-Equation (PDE, x, y, z, t: HOPS)**

Horiz. Momen.	$\frac{D\mathbf{u_h}}{Dt} + f\,\mathbf{e}_3 \wedge \mathbf{u_h} = -\frac{1}{\rho_0}\nabla_h\, p_w + \nabla\cdot(A\nabla\mathbf{u}_h)$	(1-2)
Vert. Momen.	$\rho g + \frac{\partial p_w}{\partial z} = 0$	(3)
Thermal energy	$\frac{DT}{Dt} = \nabla\cdot(K\nabla T)$	(4)
Cons. of salt	$\frac{DS}{Dt} = \nabla\cdot(K^s\nabla S)$	(5)
Cons. of mass	$\nabla\cdot\mathbf{u} = 0$	(6)
Eqn. of state	$\rho(\mathbf{r}, z, t) = \rho(T, S, p_w)$	(7)
Sound speed eqn.	$c(\mathbf{r}, z, t) = C(T, S, p_w)$	(8)

- **Acoustical model: Coupled Normal-Mode model (PDE, f, r, z, t: NPS)**

Wave eqn. $\quad \rho\, c^2(\mathbf{r}, z, t)\,\nabla\cdot\left(\frac{1}{\rho}\nabla p_s(\mathbf{r}, z, t)\right) = \frac{\partial^2 p_s(\mathbf{r},z,t)}{\partial t^2}$

p_s transfer fct. $\quad \nabla^2 P_s - \frac{1}{\rho}\nabla\rho\cdot\nabla P_s + k^2 P_s = -2\frac{r_0}{r}\,\delta(r-r_0)(z-z_0)$

where $k \doteq 2\pi f / c(\mathbf{r}, z, t)$ (9)

Coupled $\quad$ With $P_s(r, z; f) \doteq \sum_n \frac{r_0}{\sqrt{r}} P_n(r; f) Z_n(z; r, f)$ (10)

Normal-modes

$$\left\{\frac{\partial^2}{\partial z^2} - \frac{1}{\rho(r,z)}\frac{\partial\rho(r,z)}{\partial z}\frac{\partial}{\partial z} + \left(k(r,z)^2 - k_n(r;f)^2\right)\right\} \times Z_n(z; r, f) = 0 \qquad (11)$$

Modal amplit.

$$\left(\frac{d^2}{dr^2} + k_n^2\right) P_n = -\sum_m \left(\gamma_{mn}\frac{d}{dr} + C_{mn}\right) P_m \qquad (12)$$

Sonar Equation System Model (Active)

Signal-to-Noise Ratio

$$SNR = L_P - L_N = L_S - TL^A{}_1 - TL^A{}_2 + TS^T - L^N{}_n + AG + 10 \log$$

Signal-to-Interference Level

$$SIR = L_P - IL$$

Signal-to-Reverberation Ratio

$$SRR = L_P - L_R = TS^T - SS^B - 10 \log A_{BP}$$

Sonar Equation System Model (Passive)

Signal-to-Noise Ratio

$$SNR = L^T{}_S - TL^A - L^N{}_n + AG - 10 \log b - DT$$

Definitions (Active):

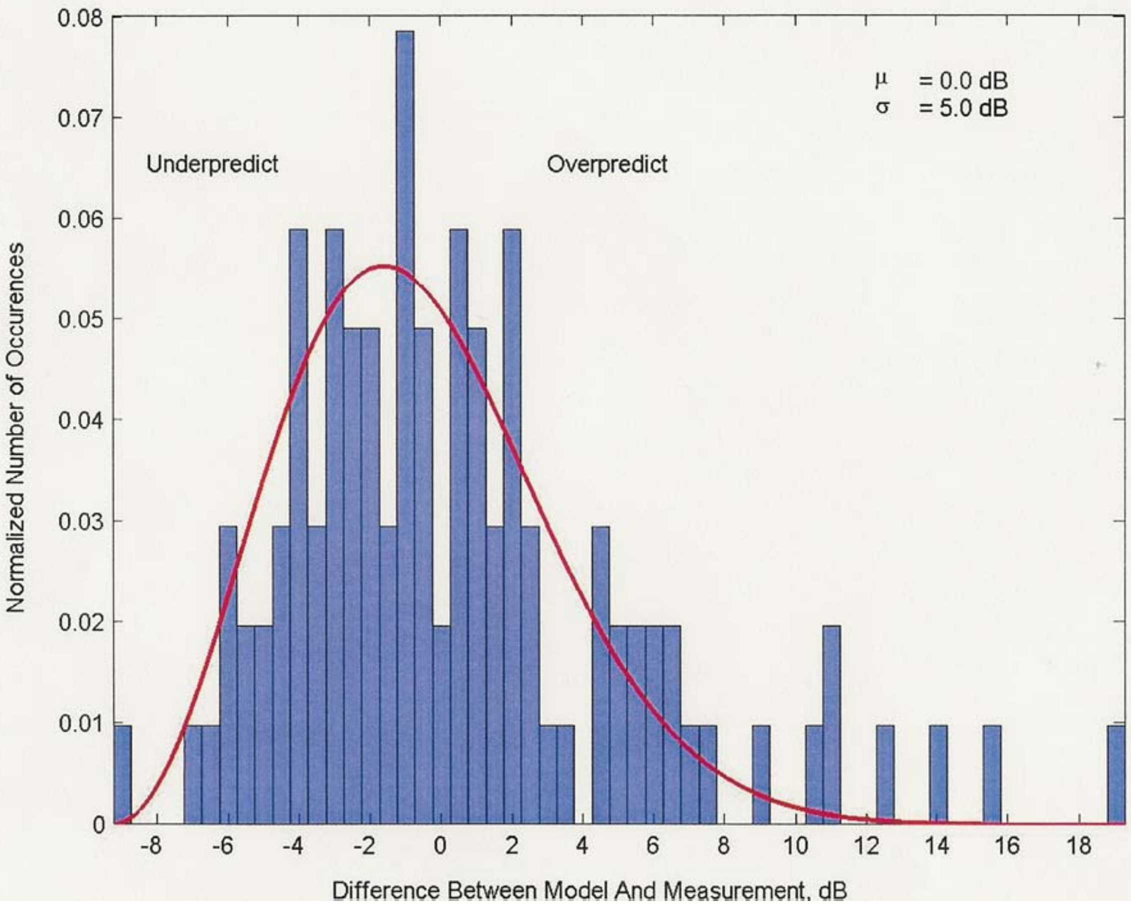
Received Level: $L_P = L_S - TL^A{}_1 - TL^A{}_2 + TS^T$
Reverberation Level: $L_R = L_S - TL^A{}_{1B} - TL^A{}_{2B} + TS_{BP}$
Noise Level: $L_N = L_n - AG - 10 \log \tau_S$
Interference Level: $IL = L_R \oplus L_N$
$\oplus$ = power sum

L_S = source level
$TL^A{}_1$ = transmission loss, source to target
$TL^A{}_2$ = transmission loss, target to receiver
TS^T = target strength
$TL^A{}_{1B}$ = transmission loss, source to bottom patch
$TL^A{}_{2B}$ = transmission loss, bottom patch to receiver
$TS_{BP} = SS^B + 10 \log A_{BP}$
SS^B = bottom scattering strength
A_{BP} = area of bottom patch
τ_S = receiver integration time

Definitions (Passive):

Received Level: $L_P = L^T{}_S - TL^A$
Noise Level: $L_N = L^N{}_n - AG + 10 \log b$
$L^T{}_S$: Source Level of Target
TL^A: Transmission Loss from Target to Receiver System
$L^N{}_n$ = Ambient noise level
AG: Receive System Array Gain
b: Receive System processing system bandwidth
DT: Receive System Detection Threshold

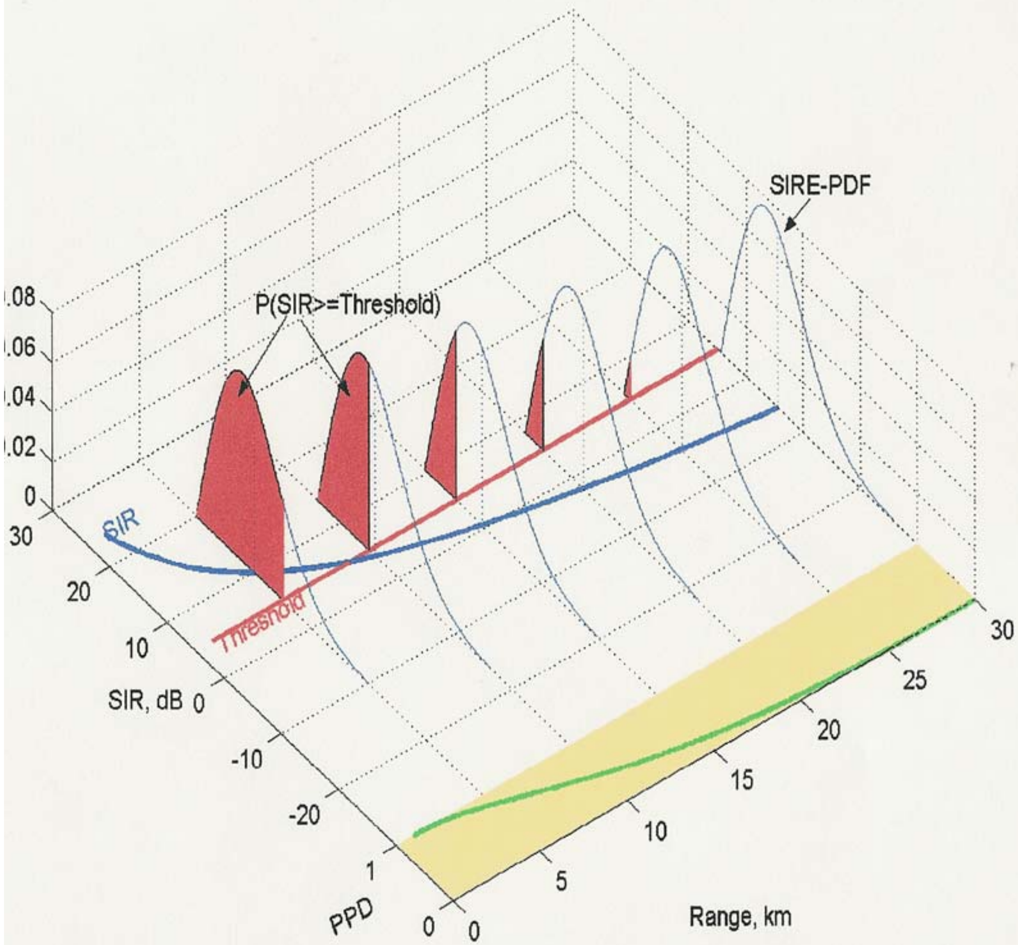
Histogram of Difference Between Model and Measured SIR, SIRE–PDF



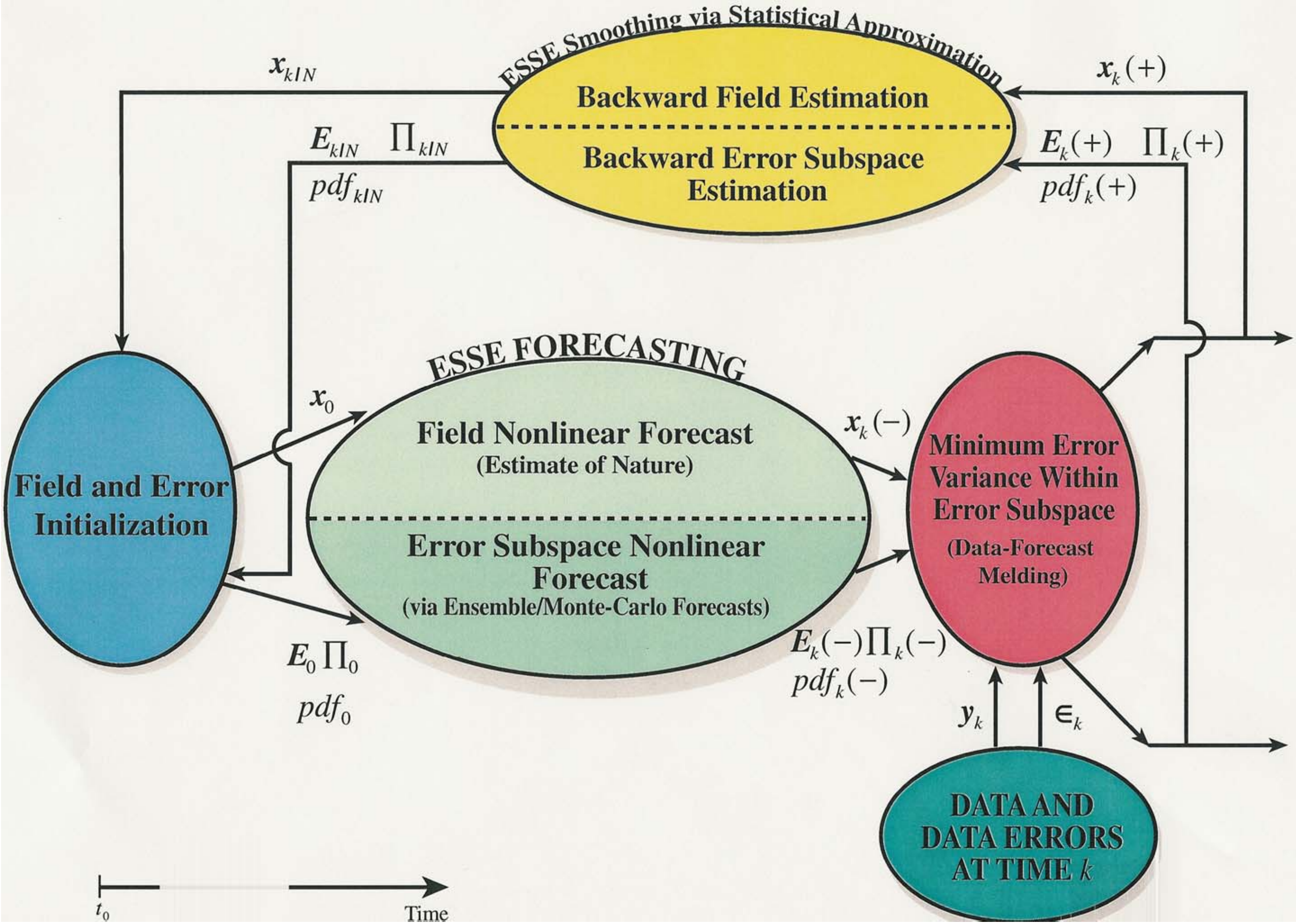
- Represents uncertainty in our ability to model actual performance of system

- Accounts for inherent variability of environment not known by current model

Determination of PPD (predictive probability of detection) using SIRE-PDF

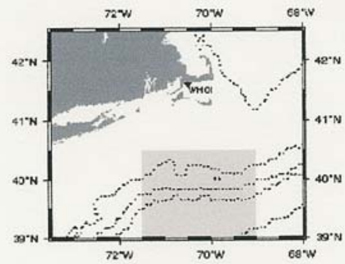


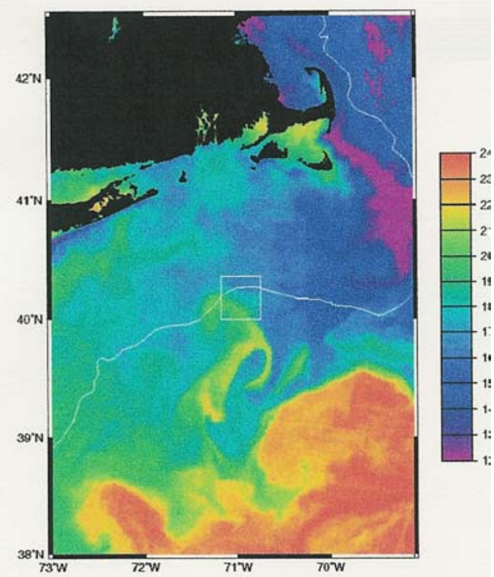
- Probabilistic representation of system performance
- Used by UNITES to characterize and transfer uncertainty from environment through end-to-end problems

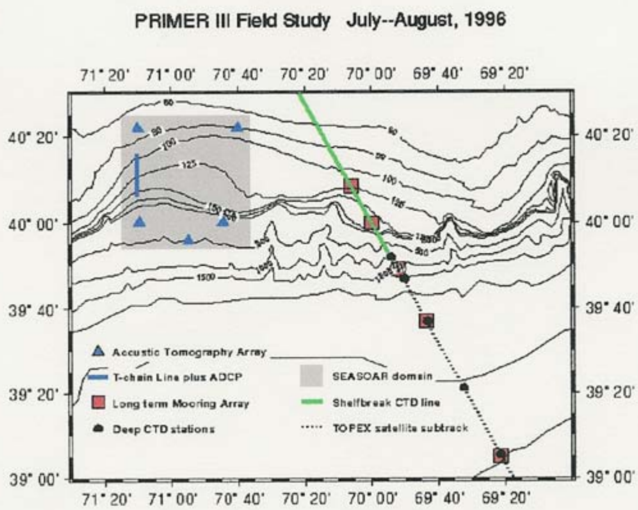

ESSE Smoothing via Statistical Approximation
Backward Field Estimation
Backward Error Subspace Estimation
$x_{k|N}$
$E_{k|N}$ $\Pi_{k|N}$
$pdf_{k|N}$
$x_k(+)$
$E_k(+)$ $\Pi_k(+)$
$pdf_k(+)$
ESSE FORECASTING
Field Nonlinear Forecast
(Estimate of Nature)
Error Subspace Nonlinear Forecast
(via Ensemble/Monte-Carlo Forecasts)
Field and Error Initialization
x_0
E_0 Π_0
pdf_0
$x_k(-)$
$E_k(-)\Pi_k(-)$
$pdf_k(-)$
Minimum Error Variance Within Error Subspace
(Data-Forecast Melding)
y_k
ϵ_k
DATA AND DATA ERRORS AT TIME k
t_0
Time

PRIMER End-to-End Problem

Focus on Passive Sonar Problem in first year







Location: Shelfbreak PRIMER Region

Season: July-August 1996

Sonar System (Receiver): Passive Towed Array

Target: Simulated UUV (with variable source level)

Frequency range: 100 to 500 Hz

Geometries: Receiver operating on the shelf shallow water; Target operating on the shelf slope (deeper water than receiver)

PRIMER: Coupled (Dynamical) Models

PHYSICAL MODELS

- Primitive-Equation model (PDE, *x,y,z,t:* HOPS)
- Objective mappings, thermal-wind balance model (HOPS/WHOI)
- Feature models (HOPS)

PHYSICAL MODELS OUTPUTS

- *T, S* and velocity fields and parameters, sound-speed field
- Dynamical balances

ACOUSTIC PROPAGATION MODELS

- Parabolic-Equation model (*x,y,z,t*: WHOI)
- Normal-mode parabolic-equation model (*x,z,f*: NPS)
- Wave number eqn. model (*x,z,f*: OASIS)
- Ray-tracing model (CASS)

ACOUSTIC PROPAGATION MODELS OUPUTS

- Full-field T.-Loss (pressure *p*)
- Modal decomposition of *p* field.
- Processed series: arrival structures, travel times, etc.
- Beamforming outputs

BOTTOM MODELS

- Hamilton model

BOTTOM MODELS OUPUTS

- Wave-speed and attenuation coefficients

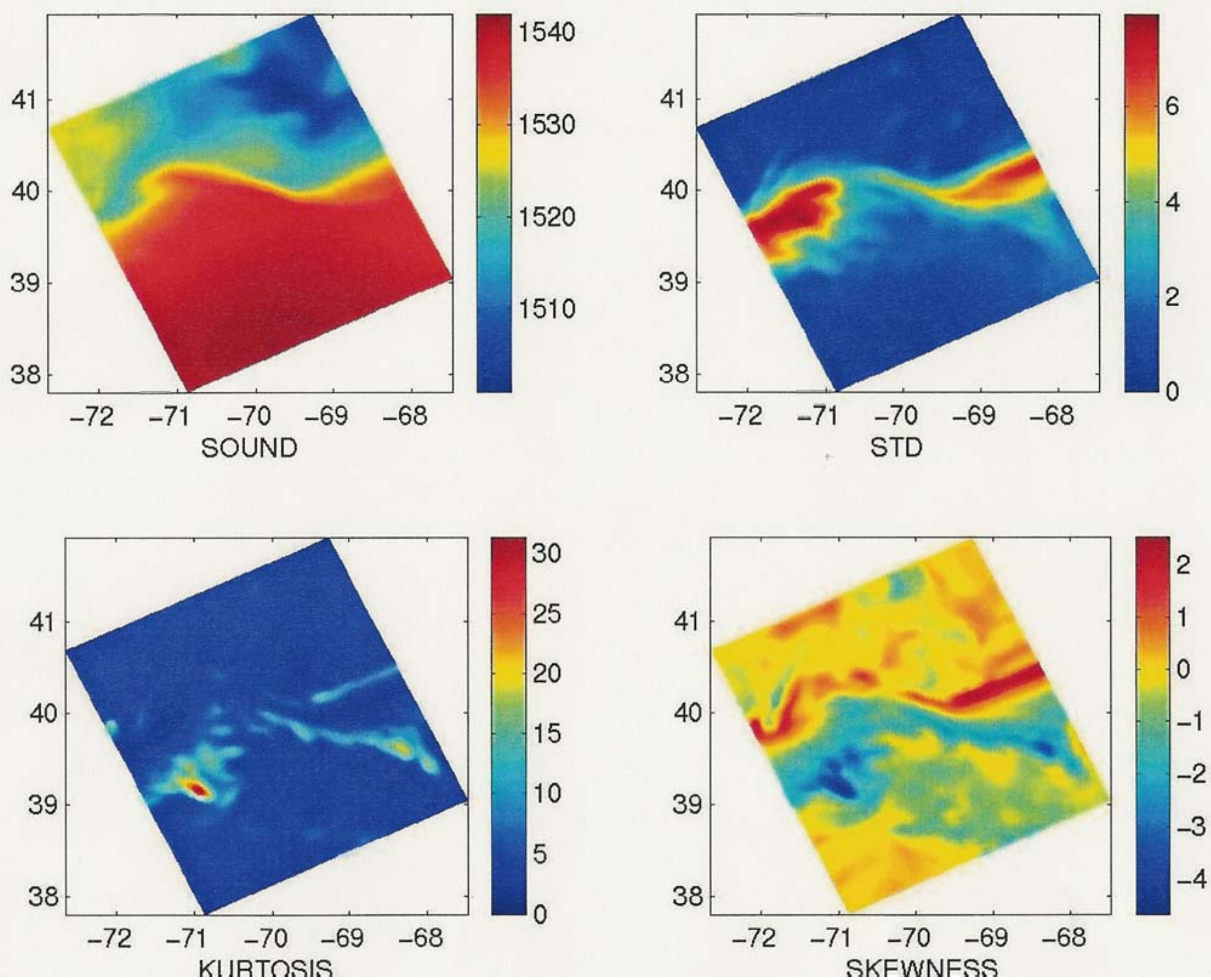
NOISE MODELS

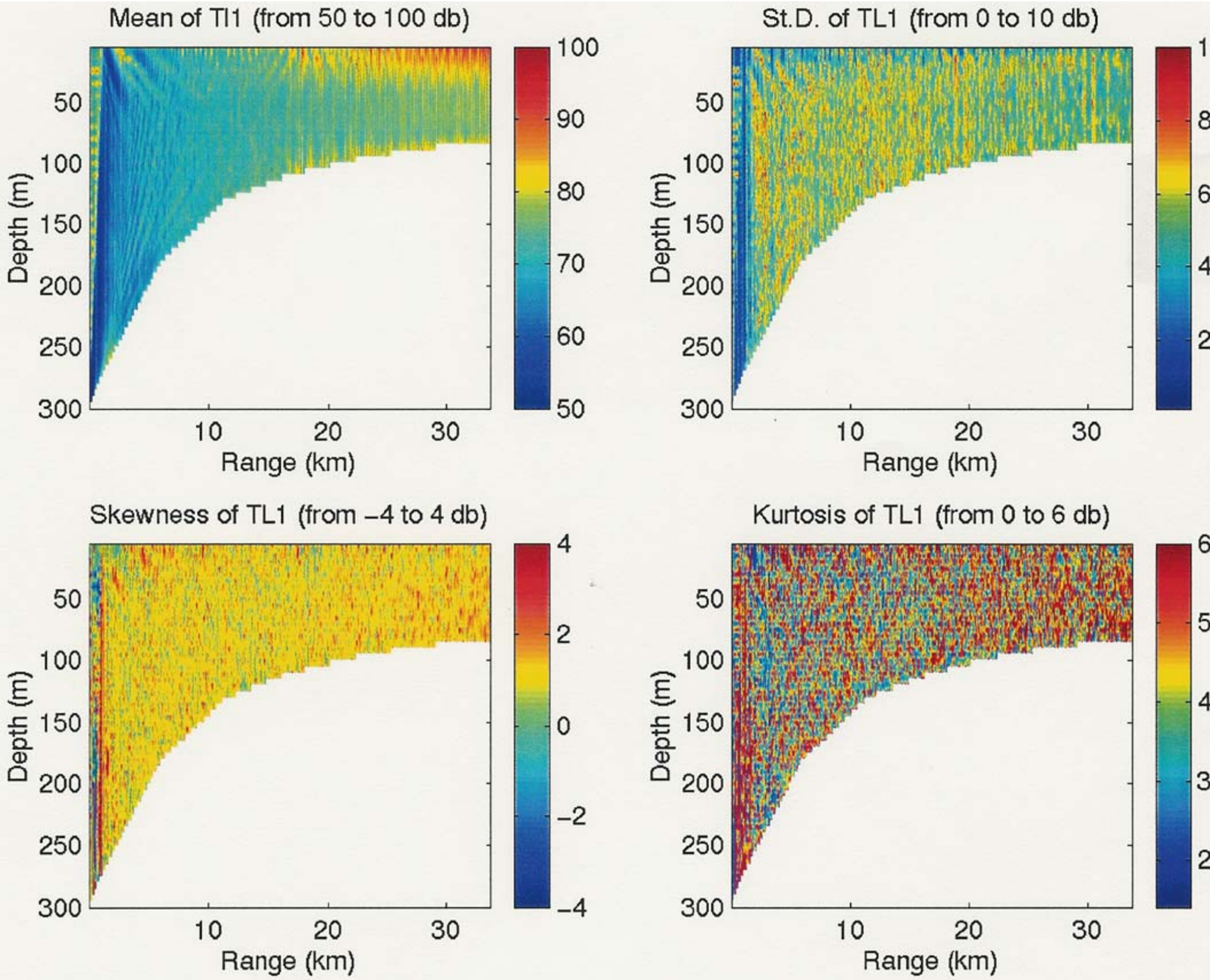
- Wenz diagram (empirical model/rule of thumb)

NOISE MODELS OUPUTS

- *f*-dependent ambient noise (*f,x,y,z,t*): due to sea-surface, shipping, biologics

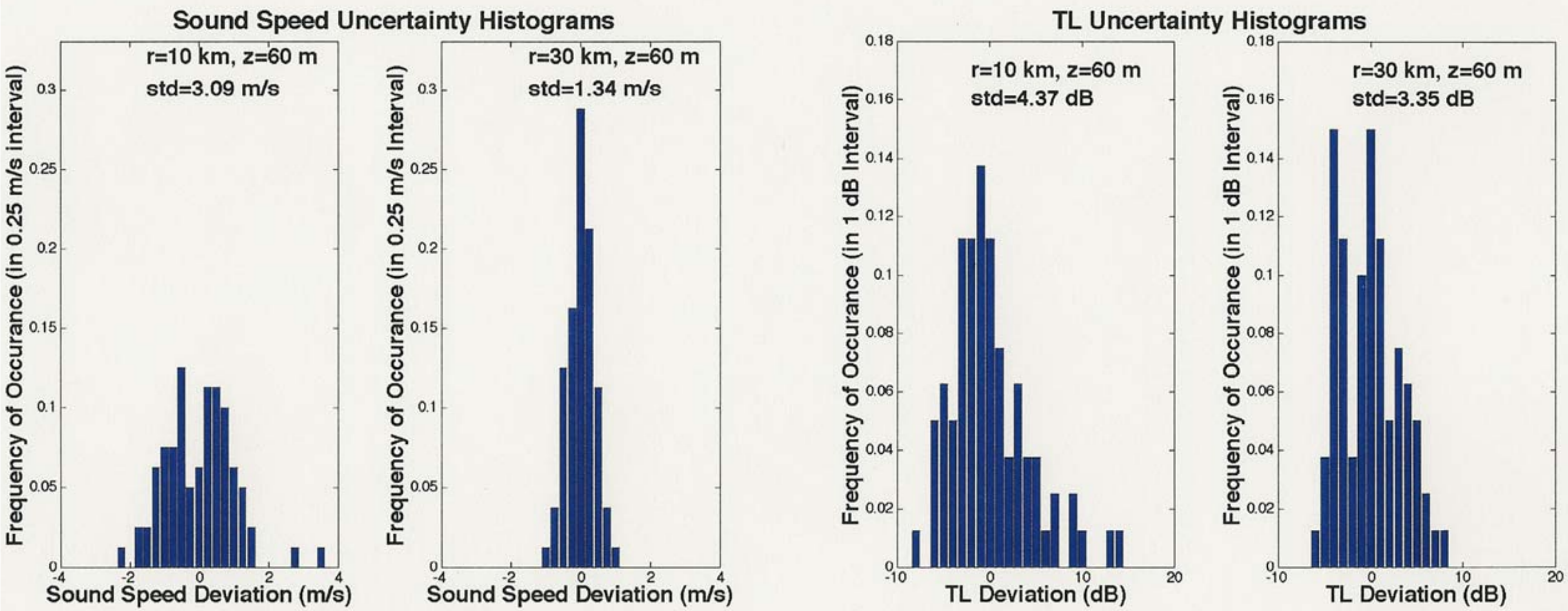
Monte Carlo simulation example: transfer of ocean physical forecast uncertainty to acoustic prediction uncertainty in a shelfbreak environment.

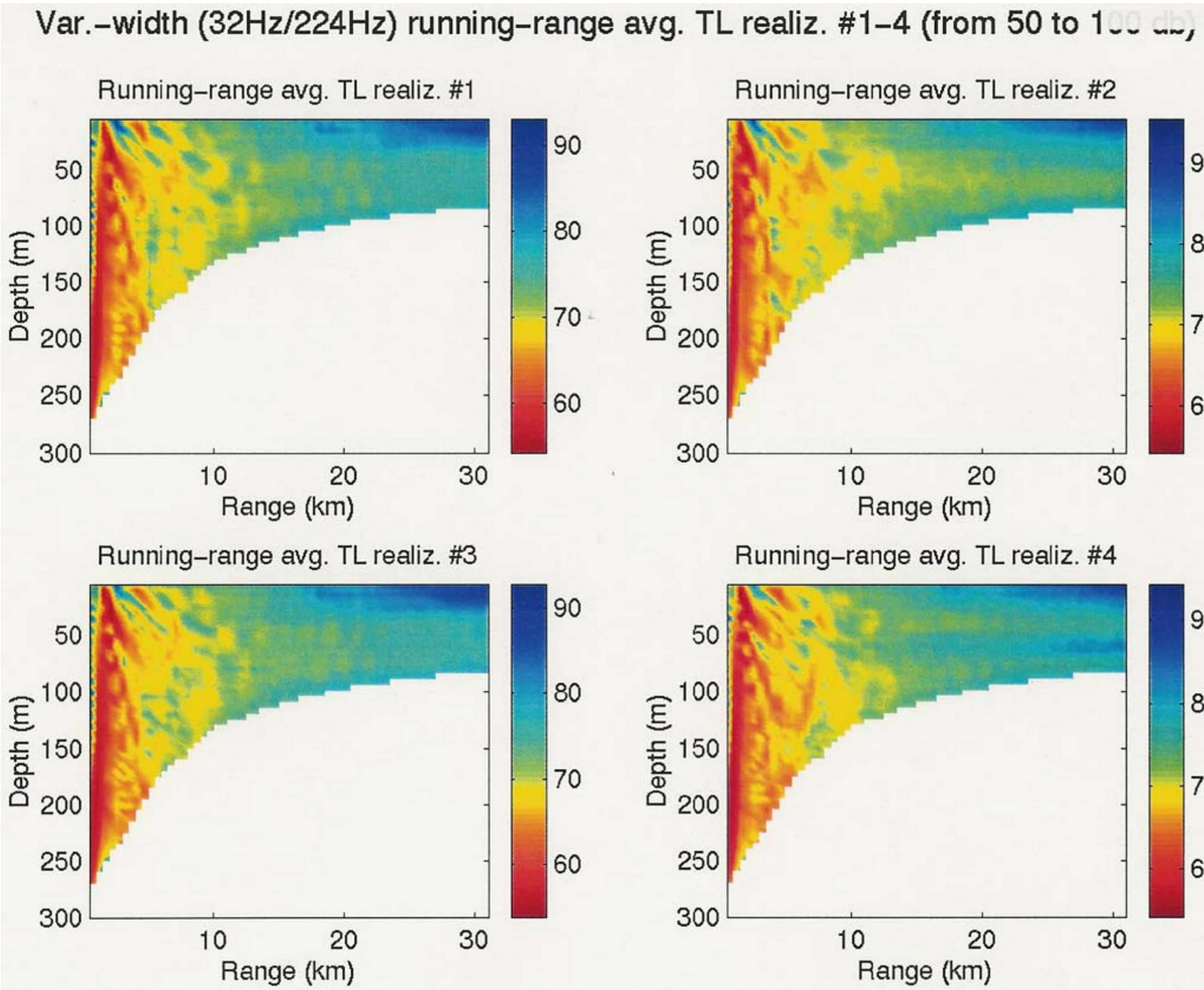




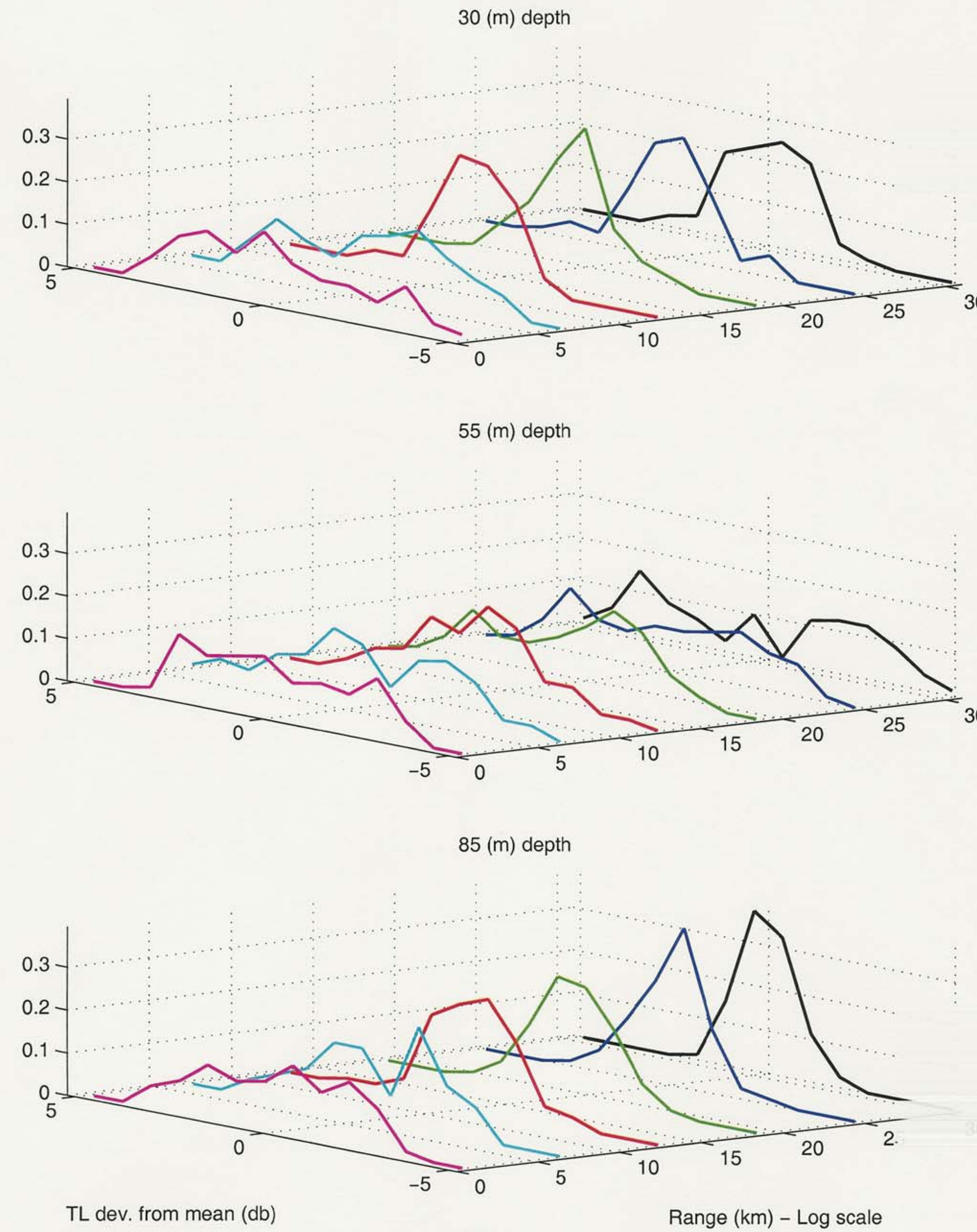
Mean and statistics of error estimate for TL1

Histograms (PDF estimates) of the sound speed and TL uncertainties at two different locations (shelfbreak and shelf).

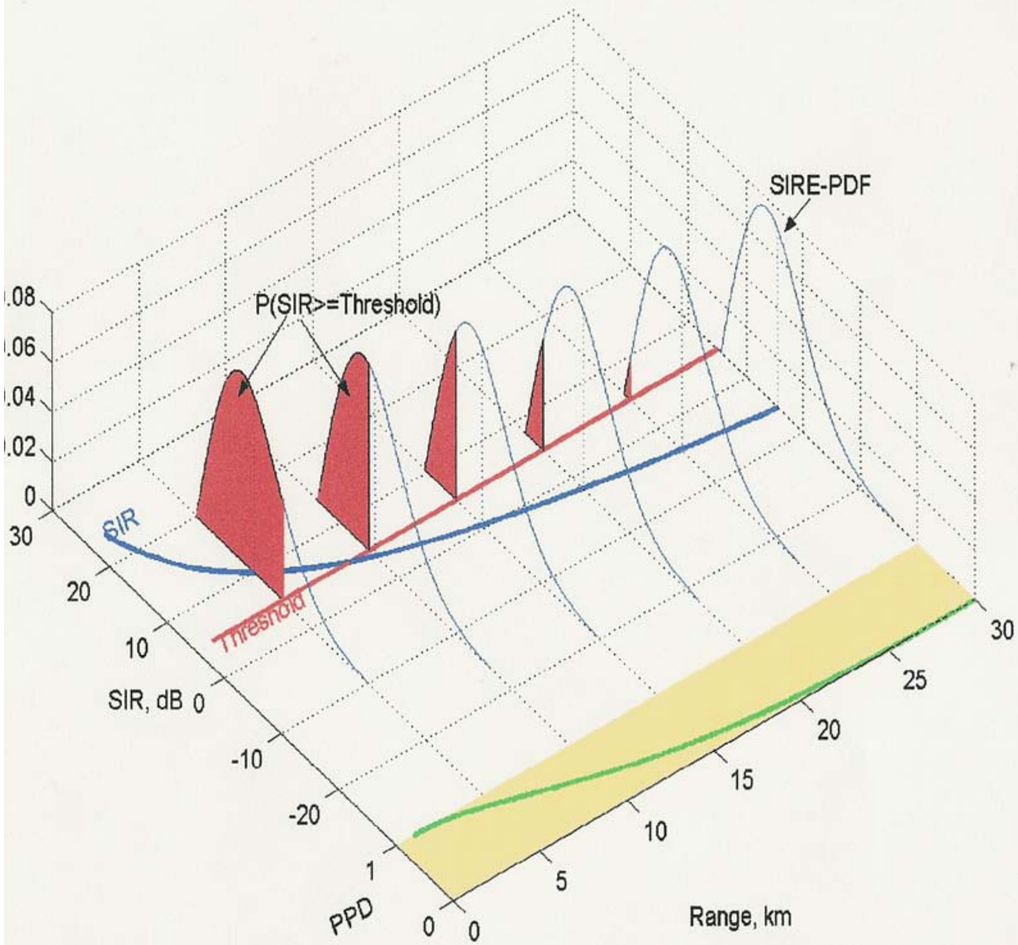



Var.–width (32Hz/224Hz) running–range avg. TL realiz. #1–4 (from 50 to 100 db)
Running–range avg. TL realiz. #1
Running–range avg. TL realiz. #2
Running–range avg. TL realiz. #3
Running–range avg. TL realiz. #4
Depth (m)
Range (km)
50
100
150
200
250
300
10
20
30
60
70
80
90

Uncertainty (error PDF) of variable-width (32Hz/224Hz) running-range avg. TL



Determination of PPD (predictive probability of detection) using SIRE-PDF



- Probabilistic representation of system performance
- Used by UNITES to characterize and transfer uncertainty from environment through end-to-end problems

HARVARD UNIVERSITY
A.R. Robinson and P.F.J. Lermusiaux

- · Formulated conceptual basis of the "end to end" system, including its components, linkages and feedbacks.
  - Generic approach for characterizing and transferring uncertainties from the ocean environment (including sea bottom) through the acoustic to the sonar and its signal processing.
-
- · Extended ESSE data assimilation to coupled physical-acoustical fields
  - Acoustic and oceanic variables both in the state vector: acoustic data modify oceanic fields and vice-versa
  - Exemplified the approach in an Identical Twin Experiment based on data collected during the summer 1996 Shelfbreak PRIMER
-
- · Initiated the use of ESSE for forecasting terms in the sonar equations that are necessary to forecast range and azimuth-dependent "predictive probabilities of detection"
-
- · Started to improve the ESSE simulations of the ocean dynamics and uncertainties during the summer 1996 Shelfbreak PRIMER. Current work involves:
  - Bottom topographies
  - Surface wind forcings
  - Numerical domain and model levels
  - Assimilation of SeaSoar data
-
- · Collected and started to study an extended bibliography of books and research and review articles on:
  - Uncertainty in ocean/atmosphere applications
  - Predictability theory and ocean/atmos./acoustic applications
  - General theories and reviews on uncertainty, information theory, etc.
  - Internal waves: data and models

List of "uncertainty, end-to-end" papers: completed and in preparation

http://www.deas.harvard.edu/~robinson
http://www.deas.harvard.edu/~pierrel

2002 - Modeling Uncertainties in the Prediction of the Acoustic Wavefield in a Shelfbreak Environment. *Proceedings of the 5th International Conference on Theoretical and Computational Acoustics*, May 21-25, 2001.
Lermusiaux, P.F.J., C.-S. Chiu and A.R. Robinson

2002 – On the mapping of multivariate geophysical fields: sensitivity to size, scales and dynamics. *J. of Atmos. Oceanic Tech.*, in press.
Lermusiaux, P.F.J.

2002 - Four-dimensional data assimilation for coupled physical-acoustical fields. To appear in the Acoustic Variability 2002 Proceedings. SACLANTCEN. Kluwer Press.
Lermusiaux, P.F.J. and C.-S. Chiu

2002 - Transfer of uncertainties through physical-acoustical-sonar end-to-end systems: A conceptual basis. To appear in the Acoustic Variability 2002 Proceedings. SACLANTCEN. Kluwer Press.
Robinson, A.R., P. Abbot, P.F.J. Lermusiaux and L. Dillman

Stochastic Modeling of Uncertainty

- Non-Gaussian sound speed error PDF's are a fundamental feature of physical acoustical models in the shelfbreak environment. Simple Gaussian approximations will be inadequate, cf. Lermusiaux et al., 2000
- Uncertainty in ocean dynamical models is transferred to acoustic models as *multiplicative* rather than *additive* noise
- Evolution of the End-to-End-System must therefore be treated in terms of the stochastic calculus; different answers will be obtained from the different formulations
- The *Stratonovich* calculus is the appropriate form for this problem
- Filtering applications may require transformation to the *Ito* formulation

Robert Miller, OSU